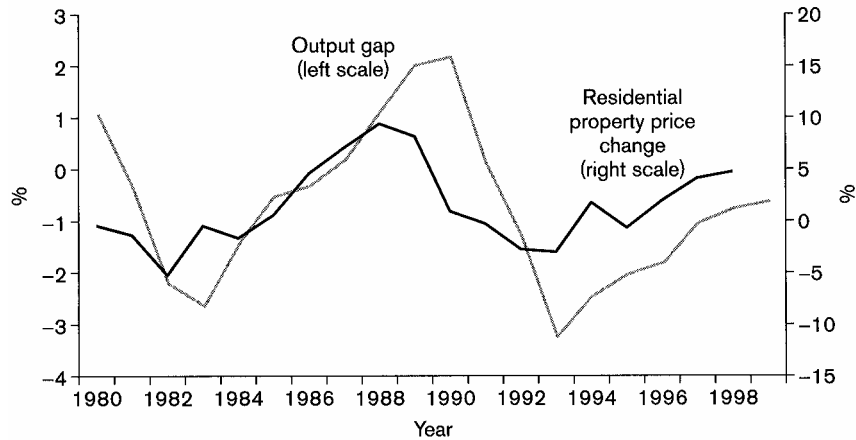


10. INVESTMENT AND ASSET PRICES

■ The previous lecture showed that **private investment is often the most volatile component of aggregate demand** and that it is **highly correlated with total output**. Understanding the forces driving **private investment is therefore crucial for understanding business cycles**.

■ In this lecture we present a **theory of business investment** as well as a **theory of housing investment**. This will give us an opportunity to study **two of the most important asset markets in the economy**: the **stock market** and the **market for owner-occupied housing**. As we shall see, **there is a systematic link between stock prices and business investment**, and a similar systematic **impact of housing prices on housing investment**. To understand investment, we must therefore study **how asset prices are formed**.

■ A glance at Figure 10.1 should make clear why we are interested in asset prices.



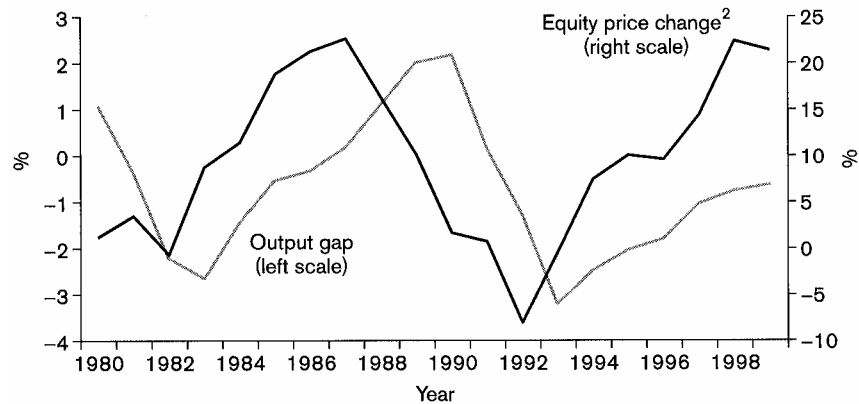


Figure 10.1: Output gap, real property prices, and real equity prices in industrial countries¹

¹ Arithmetic average of the respective variables in 16 industrial countries.

² Three-year moving average.

Source: IMF, World Economic Outlook, May 2000.

- The figure is constructed from data for the **16 most important industrial countries** and shows the **link between the evolution of housing prices and stock prices and the evolution of the output gap**. There is a **close relationship between asset price fluctuations and output fluctuations** and a clear tendency for **stock price movements to lead movements in output**.
- Thus the figure suggests that an **increase in stock prices or in housing prices will trigger an increase in economic activity**, whereas a significant **drop in asset prices may be a signal of a future economic downturn**. As we will show later, Figure 10.1 reflects that **higher asset prices tend to stimulate private consumption and investment**. In particular, the present lecture will explain **why higher stock prices tend to be followed by higher business investment**, and **why higher housing prices provide a boost to housing investment**.
- The **basic idea underlying our theory of investment** may be most easily illustrated by looking at the housing market. At any point in time there is a **certain market price for houses** of a given size and quality. This **price may well exceed the cost of constructing a**

new house of similar size and quality (the **replacement cost**). **The more the market price exceeds the replacement cost, the more profitable it will be for construction firms to build** and sell new houses. Hence we will observe a higher level of housing investment the greater the discrepancy between the market price and the replacement cost of housing.

- Note that the **market price can deviate from the replacement cost for a long time**, since it **takes time for new construction to produce a significant increase in the existing housing stock**, and since it is **time-consuming to shift economic resources into the construction industry** if construction activity becomes more profitable due to a rise in the market price of housing.

- **For business investment a similar basic principle applies.** The **market price of the business assets owned by a corporation is reflected in the market price of the shares in the firm.** The **replacement cost of the firm's assets is given by the price at which it can acquire machinery, etc., from its suppliers of capital goods.**

- **If the stock market value of the firm's assets is higher than their replacement cost, the firm can increase the wealth of its shareholders by purchasing additional capital goods, that is, by investing. The higher the stock price relative to the replacement cost, the greater is the incentive to invest, so the higher the level of investment will be.**

- **One might think that the firm would instantaneously adjust its capital stock so that any discrepancy between the stock market value and the replacement cost of its assets is immediately eliminated. However, this is not realistic, since in practice the firm will incur various costs of adjusting the capital stock, and these costs are likely to increase more than proportionally to the level of investment.**

- **Hence it will be more profitable to allow a gradual adjustment of the capital stock, and during this (potentially long) adjustment period the stock market value of the firm will deviate from the replacement cost of its assets.**

■ In the following sections we will explain this theory of the **link between asset prices and investment in more detail**, starting with the theory of the stock market and business investment.

The stock market

A few facts about the stock market

■ It is well known that **stock prices are highly volatile** and sometimes experience dramatic swings. For example, on **19 October 1987** the US Dow Jones index fell by 22.6 per cent in a single day. This was even more apocalyptic than the notorious crash in the “**Black October**” of 1929 when the Wall Street stock market dropped by 23 per cent in the course of two days (however, the **recovery of stock prices after October 1987 was much faster**, so the macroeconomic effects of the crash of 1929 were much more serious).

■ Figure 10.2 illustrates the **long-term trends in the US stock market**, documenting the evolution of **real (inflation-adjusted) stock prices** and the so-called **price-earnings ratio**,

defined as the **market value of shares relative to the profits of the companies which have issued the shares.**

■ The curve for the real stock price index highlights the **enormous stock market boom of the 1990s** which was **followed by a sharp downturn after the turn of the new century.** However, the graph for the **price-earnings ratio shows that even after this dramatic adjustment of share prices, the price of stock relative to the earnings capacity of companies remained very high** by historical standards.

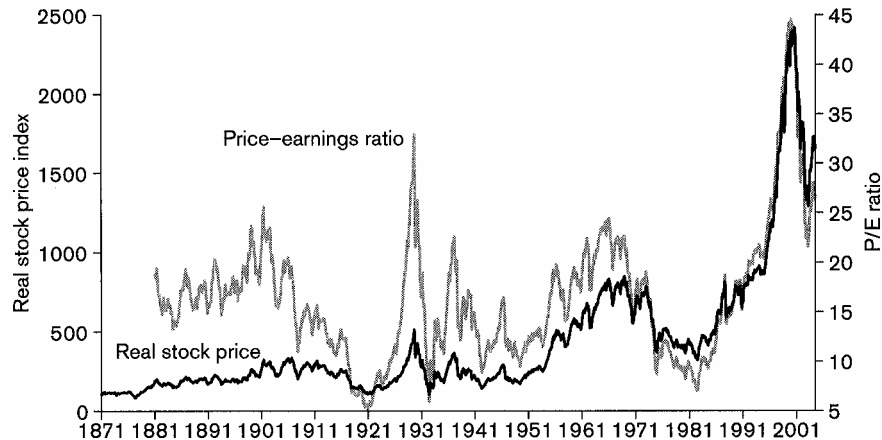


Figure 10.2: Real stock price index and the price-earnings ratio in the United States

Source: Calculated by Robert Shiller on the basis of Standard and Poor's Composite Stock Price Index, the US Consumer Price Index, and a 10-year moving average of the earnings of the companies included in the S&P stock index.

■ Mainly as a consequence of rising stock prices, the **market value of outstanding shares** (the “**stock market capitalization**”) as a percentage of GDP rose sharply during the 1990s, as shown in Figure 10.3. In this way **stocks became a much more important component of total financial wealth.**

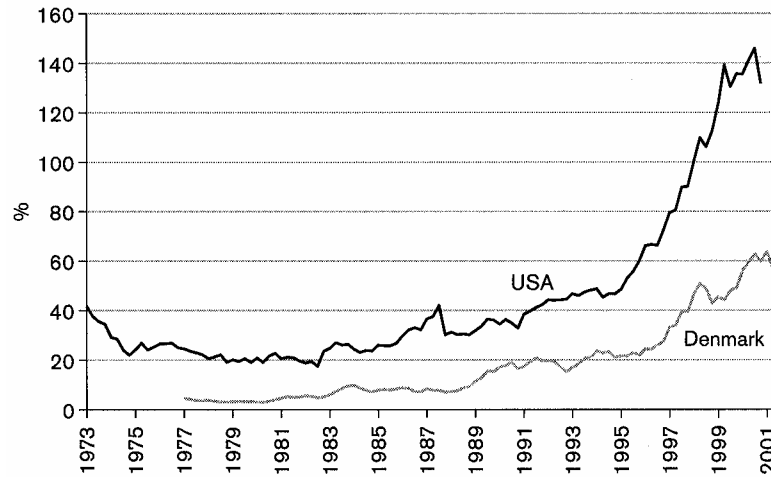


Figure 10.3: Stock market capitalization as a percentage of GDP

■ In many countries the **booming stock market motivated a growing proportion of households to invest in shares**, and **half the adult US population now owns stocks**, as indicated in Figure 10.4. In a country like Sweden, the corresponding proportion is about one third, whereas only about 15 per cent of the adult population in Denmark hold shares.

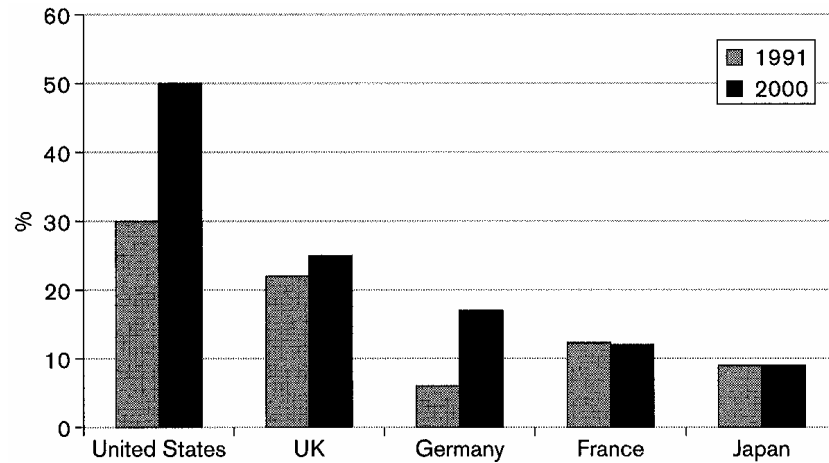


Figure 10.4: Percentage of adult population owning stocks

Source: Hali Edison and Torsten Slok, “Wealth Effects and the New Economy”, IMF Working Paper, WP/01/77, 2001.

- The **stock market is actually more important for households** than Figure 10.4 might suggest, since the **numbers in the figure only include households who are direct owners of stocks**. Households also channel a large part of their **savings through pension funds, life insurance companies** and other financial intermediaries which, in turn, invest a substantial part of their funds in shares.
- Hence the performance of the **stock market directly or indirectly determines the return to a large fraction of household savings**. In this way **stock market developments may determine when people feel they can afford to retire** from the labour market or when they can **afford to buy new consumer durables**.
- Moreover, the evolution of **stock prices may have an important impact on the level of output and employment**, because of its **influence on consumption and business investment**. In short, the **stock market is important for individual consumers and for the macro economy**, so it is worthwhile investing some effort in understanding how it works, and **how it affects investment decisions**.

The price of stock

The value of a firm and the fundamental stock price

■ The **starting point** for our analysis is the **assumption that business investment is guided by a desire to maximize the wealth of the owners** of the firm. In modern Western economies where the **bulk of business activity is carried out by firms organized as joint stock companies** (corporations), **maximization of the wealth of the firm's owners is equivalent to maximization of the market value of the outstanding shares** in the corporation. This is the **reason for our focus on the stock market**.

■ However, **our theory of investment will also be relevant for unincorporated firms** or for **corporations which are not quoted** in an official stock exchange. As we shall see, our theory of the stock market implies that the **market value of shares equals the discounted value of the expected future cashflow from the firm to its owners**.

- But this is **exactly how a rational outside investor would also value an unquoted or unincorporated firm** if he were contemplating buying or investing in such a firm. **If the owner of an unincorporated firm wants to maximize the market value of his business assets, his investment behaviour will therefore be similar to the investment behaviour of a corporation** whose shares are traded in a public stock exchange, as we shall explain in more detail later.

- You may wonder **why we assume that the objective of the firm is to maximize the wealth of its owners?** The answer is that **maximization of the current market value of the firm will also maximize the consumption possibilities of its owners.**

- This will become clear in the next lecture where we show that a **person's potential present and future consumption is constrained by the sum of his financial wealth and his current and discounted future labour income.** Therefore, if a firm can change its operations so as to increase the market value of its assets, it will increase the financial wealth of its owners and enable them to **increase their consumption either now or in the future. In both cases the owners will obviously be better off.**

- In basic microeconomics course you may have learned that **firms maximize their profits rather than the market value of their assets**. Fortunately, there is **no contradiction between these two goals**. A firm that **maximizes its discounted stream of profits over time will also maximize its market value**, as we shall demonstrate below.
- **If a corporation plans its investment with the purpose of maximizing the market value of its shares, we must base our theory of investment on a theory of the value of the firm**. Our starting point for such a theory is an **arbitrage condition** which says that the **market value of the shares in the firm must adjust to ensure that the holding of shares is equally attractive as the holding of bonds**.
- Suppose that, at the beginning of period t , the **shareholders in the firm expect to receive a dividend D_t^e at the end of the period**, and that they expect the **market value of their shares at the start of period $t + 1$ to be V_{t+1}^e** . If V_t is the **actual market value of shares in the firm at the beginning of period t** , shareholders thus expect to earn a **capital gain** equal

to $V_{t+1}^e - V_t$ during period t . Hence the **total expected return on shareholding** is $D_t^e + (V_{t+1}^e - V_t)$, composed of the **expected dividend plus the expected capital gain**.

■ **In a capital market equilibrium this expected return must equal the “required” return on shares.** The required return consists of the **opportunity cost of holding shares rather than bonds, plus an appropriate risk premium.**

■ **If r is the market rate of interest on bonds** (assumed for simplicity to be constant over time), the **opportunity cost of shareholding is rV_t** , since this is the **interest income which the shareholder could have earned** during period t if he had sold his shares at the initial market value V_t and **invested the corresponding amount in bonds.**

■ Furthermore, since **stock prices and dividends are generally more volatile than bond prices and interest payments**, shares are a **riskier investment than bonds**. Because **investors are risk averse**, the **expected rate of return on shares must therefore include a risk premium ε on top of the market interest rate** to ensure that shareholding is considered

just as attractive as the holding of bonds. Hence the **total required return on the shares** is $(r + \varepsilon)V_t$, and the **arbitrage condition for capital market equilibrium** may then be written as:

$$\overbrace{(r + \varepsilon)V_t}^{\text{required return}} = \underbrace{D_t^e + \overbrace{V_{t+1}^e - V_t}^{\text{expected capital gain}}}_{\text{total expected return on shares}} \quad 10.1$$

- **If the current market value V_t is so high that the required return on the left-hand side of (10.1) exceeds the expected return on the right-hand side, financial investors will sell off their shares** in the firm in order to buy bonds, and the **market value V_t will drop**.
- **On the other hand, if the current share price (and hence V_t) is so low that shares in the firm promise a total rate of return in excess of $r + \varepsilon$, investors will shift from bonds to shares, thereby driving up the current market value V_t .**

■ Hence the **stock market can only be in equilibrium when the arbitrage condition (10.1) is met**. Since investors derive utility from their real consumption possibilities, we are **measuring all variables in equation (10.1) in real (inflation-adjusted) terms**, so r is the **real rate of interest**. We may rearrange (10.1) to get:

$$V_t = \frac{D_t^e + V_{t+1}^e}{1 + r + \varepsilon} \quad 10.2$$

■ This is a very **important relationship** in our analysis below. It says that the **value of the firm at the beginning of any period equals the present value of that period's expected dividend plus the expected market value at the end of the period**. We see that the **rate at which future values are discounted includes the market interest rate r and the required risk premium ε** .

■ As we have argued above, the **firm will choose an investment plan which maximizes V_t** . To characterize the **firm's optimal investment policy** we must therefore **study how V_{t+1}^e and**

D_t^e and hence V_t depend on the firm's planned investment. This is the purpose of the following analysis.

■ Since **arbitrage conditions** similar to (10.2) must **hold for all subsequent periods**, rational financial investors will expect that **future stock prices will satisfy the relationships**:

$$V_{t+1}^e = \frac{D_{t+1}^e + V_{t+2}^e}{1+r+\varepsilon}, V_{t+2}^e = \frac{D_{t+2}^e + V_{t+3}^e}{1+r+\varepsilon}, \dots, V_{t+n}^e = \frac{D_{t+n}^e + V_{t+n+1}^e}{1+r+\varepsilon}, \text{ etc.} \quad 10.3$$

■ By **successive substitutions** of the expressions in (10.3) into (10.2), we find that

$$V_t = \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{V_{t+2}^e}{(1+r+\varepsilon)^2} \quad 10.4$$

$$\begin{aligned}
 &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \frac{V_{t+3}^e}{(1+r+\varepsilon)^3} \\
 &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \dots + \frac{V_{t+n}^e}{(1+r+\varepsilon)^n}
 \end{aligned}$$

■ It is reasonable to assume that **investors do not expect future real stock prices V_{t+n}^e to rise indefinitely at a rate faster than $r + \varepsilon$** , for if the opposite were the case, the current stock price V_t would become infinitely high according to (10.4). Hence we assume that

$$\lim_{n \rightarrow \infty} \frac{V_{t+n}^e}{(1+r+\varepsilon)^n} = 0 \tag{10.5}$$

■ If we continue the successive substitutions indicated in (10.4) and use our assumption (10.5), we **end up with**:

$$V_t = \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \dots \quad 10.6$$

■ Equation (10.6) is our **first important result**, stating that the **market value of the shares in a firm equals the present discounted value of the expected future dividends** paid out by the firm. This is sometimes referred to as the **fundamental share price**, because it is a price based on a “fundamental” condition, namely the **firm's ability to generate future cash flows** to its owners.

■ Note that there **must be a close correlation between a firm's dividends and its profits**, since the **latter are the source of the former**. This observation is the basis for our earlier claim that **maximization of (the present value of) profits is roughly equivalent to maximization of market value**.

Why are stock prices so volatile?

- As illustrated in Figure 10.2, **stock prices fluctuate quite a lot**. Equation (10.6) suggests **three possible explanations** for the observed volatility of stock prices:
 1. Fluctuations in (the growth rate of) **expected future real dividends**
 2. Fluctuations in the **real interest rate r**
 3. Fluctuations in the required **risk premium on shares, ε** .

- The great stock market **boom of the 1990s** seems to have been **driven mainly by more optimistic expectations regarding future real dividends**, as financial investors came to believe that the **rapid innovations in information technology would create a “New Economy”** characterized by a significantly higher real growth rate in output and business profits.

- Perhaps changes in the **required risk premium on shares** have also contributed to the turbulence on the stock market. There is some evidence that **investors frequently change their attitude towards risk**. This is illustrated by Figure 10.5 which plots recent values of a **“risk appetite index”** for the major industrial and emerging market economies, constructed by the International Monetary Fund.

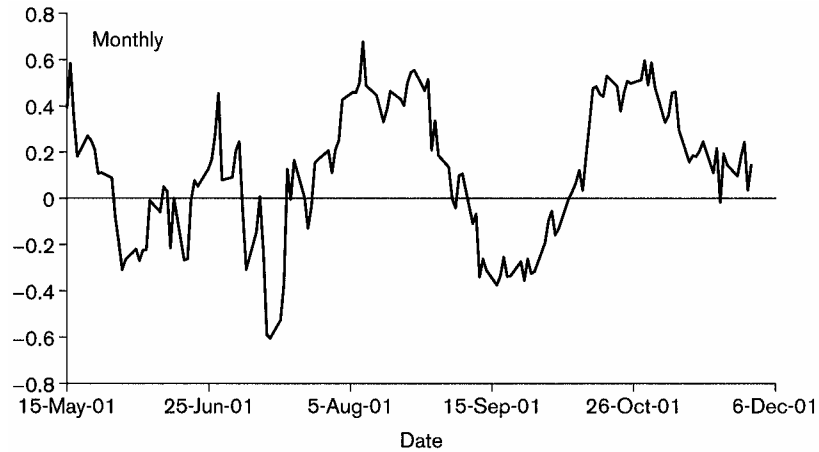


Figure 10.5: Monthly risk appetite index

Source: IMF, *World Economic Outlook*, December 2001.

- The idea behind the index is that **if investors become willing to bear more risks, they will bid up the price of assets that have been risky in the past, and if they become more risk averse they will drive down the price of risky assets by selling them.**

- Hence Figure 10.5 assumes that **movements in current asset prices which are systematically correlated with the observed past riskiness (the past volatility of returns) of those assets indicate a change in investor appetite for risk.**

- The figure suggests that the terrorist attacks on **11 September 2001 caused a sharp temporary drop in investor appetite for risk.** Following a recovery, risk appetite again began to weaken towards the end of 2001, perhaps as a reaction to the worsening news about the state of the world economy announced at that time.

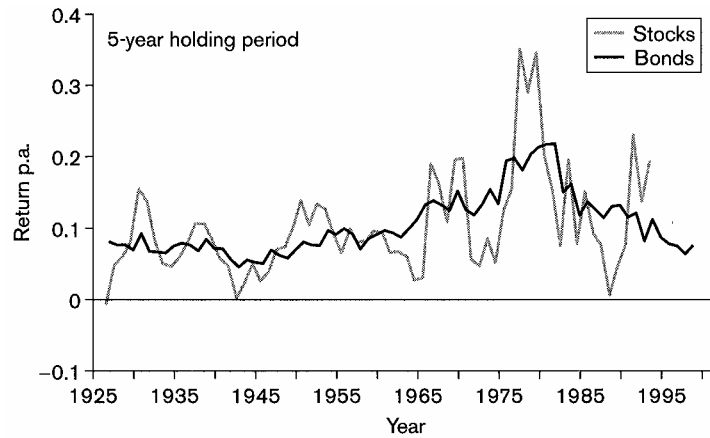
- It is often asked **whether the observed movements of the stock market are really consistent with rational investor behaviour. In the short run the stock market sometimes seems to overreact to news, showing signs of “herd behaviour”.** One may also wonder **how the large longer-term stock market fluctuations** observed during the last

decade **can be reconciled with realistic changes in the long-term earnings potential of firms**. (For a fascinating account of the less rational aspects of stock market behaviour, placing the bull market of the 1990s in historical perspective, see Robert J. **Shiller**, *Irrational Exuberance*, 2nd ed., Princeton University Press, 2005).

■ Notice, however, that the **theory presented above is compatible with the observation of frequent changes in stock prices**, if **investors frequently revise their forecasts of future dividends and their appetite for risk**, and if they are often faced with **unanticipated changes in real interest rates**.

■ Notice also that our **theory only assumes a weak form of rationality**: all we assume is that **investors require the holding of shares to be just as attractive as the holding of interest-bearing assets**. We have **not excluded the possibility that financial investors may at times hold unduly optimistic or pessimistic expectations about future dividends**, and that they may sometimes require **“unreasonably” high or low risk premia** due to an **inability to make realistic forecasts of the riskiness** of business investment.

■ In short, Eq. (10.6) makes no assumptions regarding the formation of expectations and risk premia; it only assumes that the **expected return on shares is systematically related to the return obtainable on bonds**. According to Figure 10.6 this does not appear to be a bad assumption for the longer run, since the **realized returns on bonds and stocks** (in Denmark) do indeed **seem to move together** when the rates of return are calculated over a **five-year or a ten-year holding period**.



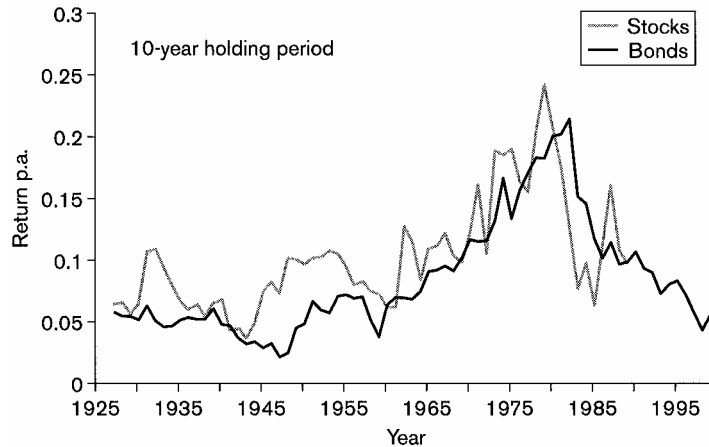


Figure 10.6: Rates of return on stocks and bonds in Denmark

Source: Jan Overgaard Olesen, "Empirical Studies of Price Behaviour in the Danish Stock Market", PhD dissertation, PhD series 2.2001, Copenhagen Business School.

- Although Figure 10.6 suggests that the **price of stocks is in fact linked to fundamentals in the long run**, many observers of the stock market believe that **stock prices can sometimes deviate from the fundamental value of firms**. During such periods of “**speculative bubbles**”, stocks become objects of **pure short-term speculation**, and their **prices cease to be pinned down by the discounted value of expected future dividends**.
- We do not deny that **speculative bubbles may sometimes occur**, but **in this lecture we will assume that stock prices reflect fundamental values** and show that the theory of the stock market outlined above can take us a long way towards understanding business investment.

Business investment

Stock prices and investment

- Our **working hypothesis** is that the **firm chooses its level of investment with the purpose of maximizing its market value V_t** . From Eq. (10.2) we see that **maximization of V_t is**

equivalent to maximizing the sum of its owners' expected dividends and expected end-of-period wealth, $D_t^e + V_{t+1}^e$, since the individual **firm has no influence on r and ε** . The question is: **what level of investment will maximize $D_t^e + V_{t+1}^e$?**

■ To answer this, let us introduce the **variable q** to indicate the **ratio between the market value and the replacement value of the firm's capital stock**. By definition, we thus have $V_t \equiv q_t K_t$, where K_t is the **real capital stock**, and where the **acquisition price of a unit of capital has been normalized to 1** so that the **replacement value of the capital stock is simply K_t** .

■ Note the **direct link between stock prices and our q -variable: if the market price of shares in the firm goes up, the value of q increases correspondingly**. The advantage of specifying our theory of investment in terms of q is that **this variable can be measured empirically** (since **stock market values** as well as **replacement values** of business assets can in principle be **observed**), whereas the **expected future dividends** underlying V and q

are very **hard to measure**. Introducing q , therefore, helps to make our **theory of investment empirically testable**.

■ Assuming that the **firm communicates its investment plans for the current period to its owners**, the **shareholders will know the size of the firm's capital stock at the start of the next period** (K_{t+1}), but they **cannot know for sure what the level of the stock price q_{t+1} at that time will be**.

■ However, **assume they expect the share price per unit of capital one period from now to be the same as the current share price** so that $q_{t+1}^e = q_t$. We then have:

$$V_{t+1}^e = q_t K_{t+1} \quad 10.7$$

- We have now **expressed** V_{t+1}^e **in a form** that will turn out to be convenient. Let us next consider the **other determinant of current market value** $V_t = (D_t^e + V_{t+1}^e)/(1 + r + \varepsilon)$, that is, the expected dividend D_t^e for period t .

- Suppose for the moment that the **firm finances all of its current investment spending I_t via retained profits** (we shall **consider external financing later on**). Furthermore, suppose realistically that **increases in the firm's capital stock imply adjustment costs**, including **costs of installing new machinery**, costs of **training workers** to use the new equipment, and possibly costs of **changing the firm's organization**. For convenience, all such costs will be called “**installation costs**” and will be denoted by $c(I_t)$ to indicate that they are a **function of investment spending**.

- With these assumptions, the **expected dividend for period t will be equal to the expected profit in period t , Π_t^e** (measured **before deduction of installation costs**), **minus that part**

of profit which is retained in order to finance the expenditure $I_t + c(I_t)$ associated with new investment:

$$D_t^e = \Pi_t^e - I_t - c(I_t), \quad c(0) = 0, \quad c' > 0 \quad 10.8$$

■ It seems reasonable to **assume that installation costs will rise more than proportionately with investment spending**. If investment is low, the changes in the capital stock are small and can be accommodated without significant changes in the firm's organization.

■ But **when investment is high, the firm may have to undertake significant organizational changes and extensive training of employees**, and the attention of managers will be diverted from the firm's day-to-day business. Such an organizational overhaul is typically very expensive. A **simple installation cost function** capturing this assumption is:

$$c(I_t) = \frac{a}{2} I_t^2 \quad 10.9$$

where a is a positive constant. Equation (10.9) implies that $dc/dI_t = aI_t$, that is, the **marginal installation cost increases in proportion to the level of investment**, reflecting that large changes in the capital stock are disproportionately more costly than small changes. To derive our investment schedule, we finally need the **identity**:

$$K_{t+1} = K_t + I_t \quad 10.10$$

stating that the capital stock at the beginning of period $t + 1$ equals the capital stock existing at the beginning of the previous period plus the level of investment during period t . For simplicity, (10.10) **abstracts from depreciation** of the existing capital stock, but our theory of investment can easily be generalized to allow for depreciation.

- Starting from (10.2), and inserting (10.7)-(10.10) into (10.2), we now find that the **firm's market value at the start of period t** can be written as follows:

$$V_t = \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon} = \frac{\overbrace{\Pi_t^e - I_t - \frac{\alpha}{2} I_t^2}^{D_t^e} + \overbrace{q_t(K_t + I_t)}^{V_{t+1}^e}}{1+r+\varepsilon} \quad 10.11$$

- The **firm chooses its level of gross investment I_t** so as to **maximize the initial wealth of its owners, V_t** , taking the **stock market's valuation of a unit of capital (q_t) as given**. The **first-order condition $\delta V_t / \delta I_t = 0$** for the solution to this maximization problem yields:

$$\underbrace{q_t}_{\text{expected capital gain}} = 1 + \frac{\overbrace{dc}_{\text{foregone dividend}}}{dI_t}$$

and then, using $dc/dI_t = aI_t$:

$$I_t = \frac{q_t - 1}{a} \quad 10.12$$

- The **investment rule** $q_t = 1 + dc/dI_t$ may be explained as follows: **to finance the acquisition and installation of an extra unit of capital in period t** , the firm must reduce its **dividend payout** in period t by an amount equal to the **acquisition cost of a unit of capital** – which we have set equal to 1 – **plus the marginal installation cost dc/dI_t** .
- This **forgone dividend** $1 + dc/dI_t$ is the **shareholder's marginal opportunity cost of allowing the firm to undertake an extra unit of investment**. The **shareholder's marginal benefit from investment** is the **gain q_t in the value of shares** resulting from the **installation of an extra unit of capital**.

- **At the optimal level of investment, the marginal dividend forgone is just compensated by the extra capital gain on shares.** Clearly, **the higher the market valuation q_t of an extra unit of capital, the further the firm can push its level of investment** before the marginal installation cost reaches the threshold where the shareholder's additional capital gain is offset by the extra dividend forgone.
- Hence we obtain the **simple investment schedule** in (10.12) which says that **investment will be higher the higher the level of the stock price q .** Equation (10.12) also shows that **high marginal installation costs** (reflected in a high value of the cost parameter a) **reduce the optimal level of investment**, as one would expect.
- This is also clear from the graphical illustration of the determination of investment in Figure 10.7: a **higher value of a increases the slope of the curve $1 + dc/dI_t = 1 + aI_t$** and thereby **reduces the value-maximizing level of investment** where $1 + dc/dI_t = q_t$.

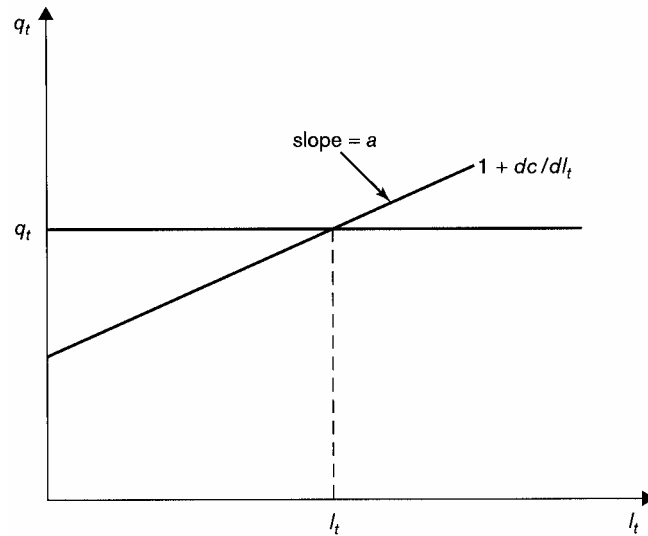


Figure 10.7: The optimal level of investment

- The **investment schedule** (10.12) also **holds when investment outlays are financed by issuing new debt or new shares** rather than by retaining profits. Regardless of the mode of finance, the **installation of an extra unit of capital will increase the expected market value of the firm's assets by the amount q_t** , still assuming that the current stock price gives an indication of the expected value.

- **If the cost of buying and installing the extra unit of capital $(1 + dc/dI_t)$ is financed by an increase in the firm's outstanding debt or by the issue of new shares**, the **expected increase in the market value of the shares** owned by the firm's existing shareholders will be **equal to the rise in total market value q_t minus the value of the newly issued debt or equity, $1 + dc/dI_t$** .

- Of course it is **optimal** for existing shareholders to let the firm **expand its investment until the expected marginal gain in the value of their shares is driven down to 0**, that is, until $q_t - (1 + dc/dI_t) = 0$. But this is **exactly the investment rule** leading to the investment

schedule in (10.12). Hence we obtain the important result that the **investment function** $I_t = (q_t - 1)a$ is valid **regardless of the method of investment finance**.

■ The theory outlined above is known as **Tobin's q -theory of investment**, named after Nobel Laureate James Tobin who was the first economist to give a systematic formal account of the link between stock prices and business investment. The classic statement of the theory was given in James **Tobin**, “A General Equilibrium Approach to Monetary Theory”, *Journal of Money, Credit, and Banking*, 1, 1969. The theory was later refined and extended by Fumio **Hayashi**, “Tobin's Marginal q and Average q : A Neoclassical Interpretation”, *Econometrica*, 50, 1982.

■ Figure 10.8 plots total **fixed business investment in the US** against an estimate of **Tobin's q** , defined as the **ratio of the stock market value of firms to the replacement value of their capital stock**, as above.

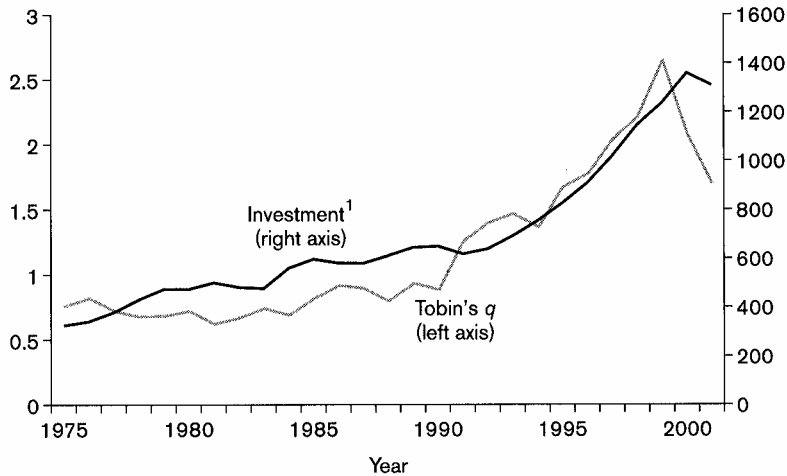


Figure 10.8: Investment and Tobin's q in the United States

Note: Real private non-residential fixed capital formation measured in 1996 US dollars.

Source: Board of Governors of Federal Reserve System (1999), Flow of Funds Accounts of the United States: Flows and Outstanding Stocks (fourth quarter 1998).

- We see that although **total investment and the q -ratio do tend to move together** over the longer run, they **do not always change in the same direction in the short run**. Part of the problem may be that, in practice, **stock prices reflect many “intangible” business assets besides physical capital**, for example **patents and know-how**.
- Another part of the explanation for the sometimes weak relationship between investment and Tobin's q may be that the **estimated value of q reflects the *average* ratio between the total market value and the total replacement value of the capital stock**, whereas in theory, **investment decisions depend on the *marginal* value of q** , that is, on the increase in market value relative to the acquisition price of an additional unit of capital.
- **In our analysis** above, the **marginal and average values of q were identical**, because our simplifying assumptions implied **proportionality between the firm's future capital stock and expected future profits**. But if this proportionality breaks down, the **marginal q will no longer coincide with the average q** , and since **only the latter can be measured empirically**, this may make it difficult to test the q -theory of investment.

The role of interest rates, profits and sales

- How does the q -theory of investment square with the conventional assumption that **investment depends negatively on the real interest rate**? The claim that investment varies positively with stock prices is fully consistent with the hypothesis that it varies negatively with the real interest rate.
- To see this, let us go back to Eq. (10.6) and let us **assume** for simplicity (since this will not affect our qualitative conclusion) **that real dividends are expected to stay constant at the level D_t^e from period t and onwards**. Equation (10.6) then becomes:

$$V_t = D_t^e \left[\frac{1}{1+r+\varepsilon} + \frac{1}{(1+r+\varepsilon)^2} + \frac{1}{(1+r+\varepsilon)^3} + \dots \right] \quad 10.13$$

■ If we **multiply both sides** of (10.13) by $1 + r + \varepsilon$ and **subtract** (10.13) from the resulting equation, we get:

$$V_t = \frac{D_t^e}{r + \varepsilon} \quad 10.14$$

■ Equation (10.14) is just a **special version of the general formula** stating that the **value of the firm equals the present discounted value of expected future dividends**. Now recall that, by definition, $V_t = q_t K_t$. From this relationship and (10.14) it follows that:

$$q_t = \frac{D_t^e / K_t}{r + \varepsilon} \quad 10.15$$

■ According to (10.15) the **market value of a unit of the firm's capital stock (q_t) equals the discounted value of the expected future dividends per unit of capital**.

- Hence a **rise in the real interest rate r** will, *ceteris paribus*, **reduce the stock price q_t** , and this will **reduce investment**. From Figure 10.6 and Figure 10.8 we have seen that **stock prices do tend to adjust to keep the return on stocks in line with the return on bonds**, and that **investment tends to move in line with stock prices**. This is **indirect evidence of a negative impact of interest rates on investment**.
- Figure 10.9 shows direct evidence of the **relationship between the bond interest rate and the ratio of business investment to output** in Denmark. (Ideally, Figure 10.9 should plot the real interest rate, but since the expected inflation rate is unobservable, we have pragmatically chosen to show the nominal interest rate as a proxy for the real rate.)

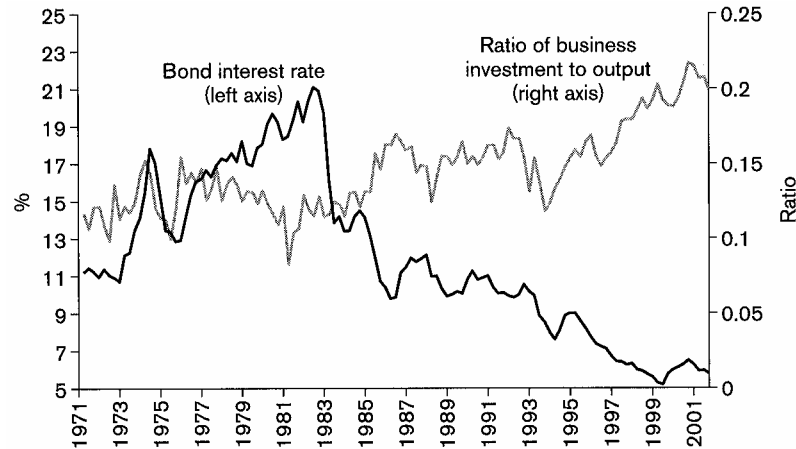


Figure 10.9: Investment ratio and bond interest rate in Denmark

Source: MONA database, Danmarks Nationalbank.

- We see that there has been a **clear negative relationship between the interest rate and investment** in the second half of the 1990s. In other periods the negative correlation has been less clear, but this is not surprising, since our Eq. (10.15) implies that **q will fluctuate not only with interest rates, but also with changes in the risk premium ε and in the expected dividend ratio, D^e/K** . Let us take a closer look at the likely **determinants of the latter variable**.

- It seems reasonable to **assume that expected dividends are positively related to the observed current profits** of the firm, Π_t . For concreteness, suppose **shareholders expect that the firm will pay out a fraction θ of its profits as dividends** at the end of the period, so $D_t^e = \theta\Pi_t$.

- In that case the numerator of (10.15) may be written as $\theta(\Pi_t/K_t)$, where Π_t/K_t is the **firm's current rate of profit** (the profit to capital ratio). From (10.15) we would then **expect to observe a positive correlation between (changes in) the current profit rate and (changes in) current investment**. Figure 10.10 suggests that such a positive relationship does in fact exist.

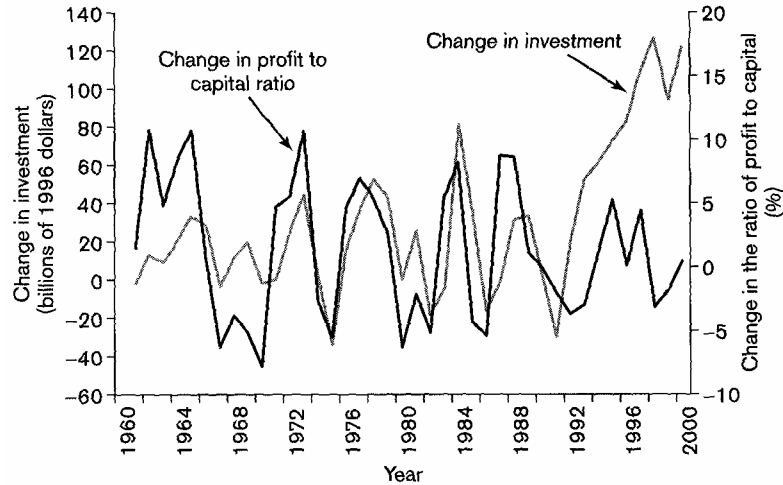


Figure 10.10: Changes in investment and profit in the United States 1960-2000

Source: National Income and Products Accounts (Bureau of Economic Analysis).

- We would also expect a positive relationship between the profit rate and the output-capital ratio. For example, we know from growth theory that if output Y is given by the Cobb-Douglas production function $Y = AK^\alpha L^{1-\alpha}$ (where L is labour input), and if markets are competitive, total profits will be equal to αY (see Lecture 3).
- In that case the rate of profit is $\alpha Y/K$ which is directly proportional to the output-capital ratio Y/K . Even if markets are not competitive, it is still reasonable to assume that the more firms can produce and sell on the basis of a given capital stock, the higher their profit rate will be. Figure 10.11 roughly confirms this expectation.

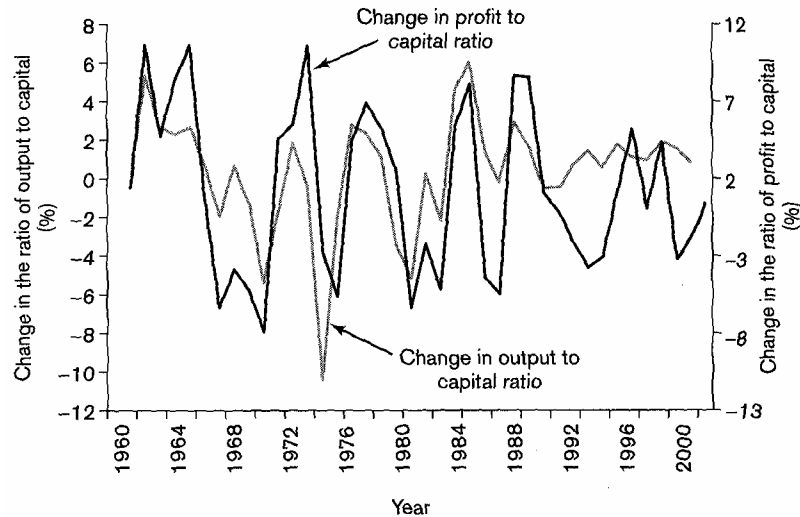


Figure 10.11: Changes in the ratio of output to capital and profit to capital in the United States 1960-2000

Source: National Income and Products Accounts (Bureau of Economic Analysis).

- Although a **higher current profit is likely to boost expected future dividends**, it is too **primitive to expect a mechanical one-to-one impact** of the former on the latter variable. Firms and **investors may sometimes have good reason to expect that future profitability will deviate from realized current profits**.
- Indeed, the fact that the **economy moves up and down in cycles** suggests that **intelligent investors will not mechanically extrapolate current earnings into the future**. Instead they will **revise their expectations regarding future sales and profits** as they **receive new information** on relevant economic and political events. Most advanced countries publish indices of “**business confidence**” to measure business expectations regarding the near future.
- Usually these indices build on **survey data** where a sample of business **managers report current and expected movements in their future output, sales, employment, investment**, etc. Figure 10.12 shows the evolution of one such index of business confidence in the US. We see that **business confidence can fluctuate quite a lot** and that it is sometimes significantly affected by **unanticipated political events**.

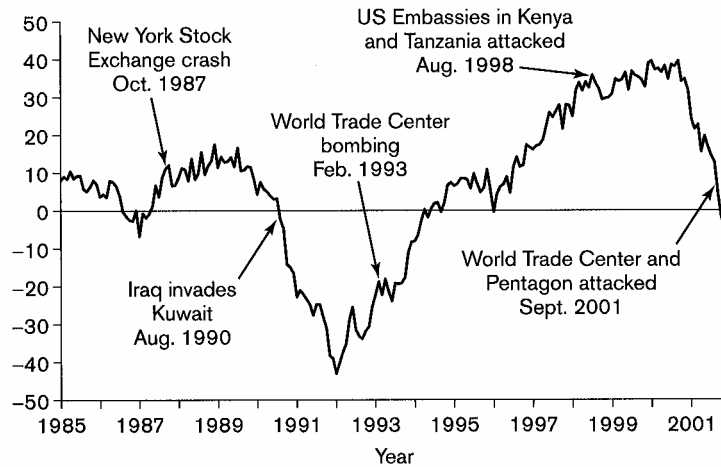


Figure 10.12: Business confidence in the United States

Source: IMF, World Economic Outlook, December 2001.

■ Hence, to understand the high volatility of investment spending, we have to allow for changes in expectations regarding the future. Our earlier discussion of Figure 10.5 suggests that part of the impact of a change in expectations on investment may be caused by a change in the required risk premium ε .

■ To summarize, we have seen that the *q*-theory of investment is quite consistent with the hypothesis that business investment varies positively with the level of output (sales), whereas the existing capital stock and the real interest rate both have a negative impact on investment. Specifically, if E is an index of the “state of confidence”, we have argued that the expected dividend ratio D_t^e/K_t will be given by a function like $g(Y_t/K_t, E_t)$, where both of the first derivatives of the $g()$ -function are positive. Using this relationship along with (10.12) and (10.15), we may thus write our investment function as:

$$I_t = \left(\frac{1}{a}\right) \left(\frac{D_t^e/K_t}{r + \varepsilon} - 1 \right) = \left(\frac{1}{a}\right) \left(\frac{g(Y_t/K_t, E_t)}{r + \varepsilon} - 1 \right)$$

or, in more general form, and **dropping time subscripts for convenience**:

$$I = f\left(\underset{(+)}{Y}, \underset{(-)}{K}, \underset{(-)}{r}, \underset{(+)}{E}\right) \quad 10.16$$

where the signs below the variables indicate the **signs of the corresponding partial derivatives** of the $f()$ -function.

■ In terms of the q -theory, an **increase in Y or E will stimulate investment by raising q_t through an increase in the expected dividend ratio $D_t^e/K_t = \theta g(Y_t/K_t, E_t)$ (and possibly through a fall in the risk premium ε). An increase in the current capital stock K reduces investment by driving down D_t^e/K_t , and an increase in the real interest rate r likewise discourages investment via a negative impact on q_t .**

■ In the case of firms whose market value is not directly observable because they are not quoted on the stock exchange, it is inappropriate to interpret Eq. (10.16) literally in terms of

the q -theory. Nevertheless, the **investment behaviour of such firms may still be described by an equation like (10.16)** if they invest with the purpose of **maximizing the present value of the net cash flow to their owner** (thereby **maximizing his wealth**). Equation (10.16) therefore summarizes our general theory of business investment.

■ **Econometric research has confirmed that changes in Y , K and r influence investment in the manner indicated in (10.16).** However, researchers have also found that it is **quite difficult to explain fully all of the observed movements in investment**. To illustrate, Figure 10.13 plots **actual investment against the predicted level** of investment estimated on the basis of a sophisticated version of the investment function (10.16).

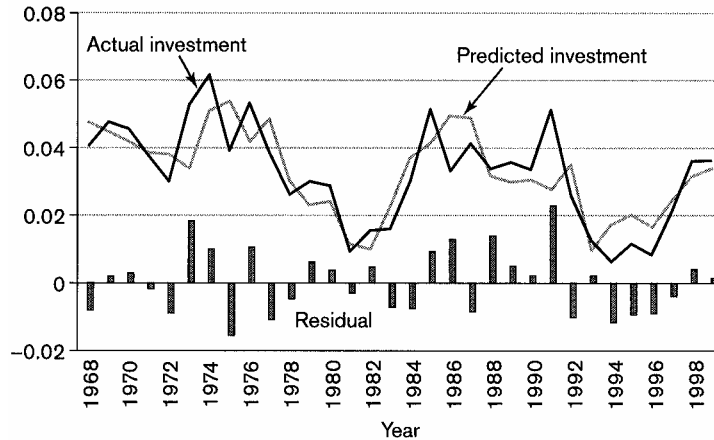


Figure 10.13: Actual and predicted values of net investment ratio in Denmark

Note: Net business investment in machinery and equipment relative to value added.

Source: SMEC database, Secretariat of the Danish Economic Council.

■ To a large extent, the **difficulties of predicting investment** undoubtedly **stem from the difficulty of finding reliable quantifiable proxies for “the state of confidence”**, *E*. Because **expectations are so hard to measure** and may sometimes **change abruptly**, it is inherently difficult to forecast investment.

The housing market and housing investment

A *q*-theory of housing investment

■ **Housing investment is an important component of total private investment**, and as indicated in Table 15.1, it is **even more volatile than business investment**.

Table 10.1: The volatility of housing investment

	Average share of total private investment (%)	Coefficient of correlation with contemporaneous GDP (same quarter)	Absolute volatility (standard deviation)	Volatility relative to volatility of total private fixed investment	Volatility relative to volatility of GDP
Belgium	29	0.47	6.15	1.37	5.78
Denmark	28	0.70	9.57	1.20	5.72
Netherlands	32	0.22	5.95	1.47	4.87
United Kingdom	25	0.47	8.78	1.82	5.81
United States	31	0.54	10.19	2.12	6.13

Sources: UK Office for National Statistics, Ecowin, Datastream, Bureau of Economic Analysis, Bureau of Labor Statistics, Federal Reserve Bank of St. Louis, Danmarks Nationalbank, Belgostat, Banque Nationale de Belgique, Statistics Netherlands and De Nederlandse Bank.

■ Hence **fluctuations in residential investment often play an important role during business cycles**. A basic factor contributing to the volatility of housing investment is the fact that **housing capital is highly durable**. In any year the **construction of new housing is only a very small fraction of the existing housing stock**. To accommodate even small

percentage changes in consumer demand for housing capital, construction activity may therefore have to undergo large relative changes.

■ In this section we will show that **housing investment may be explained along lines which are similar to the q -theory of business investment.** The present section may therefore be seen as an illustrative **special version of the q -theory**, adapted to fit the housing market.

■ As a **by-product of our theory of housing investment**, we will develop a **theory of the formation of housing prices** and identify the **factors which may cause fluctuations in the market value of the housing stock.**

■ Since the **stock of housing capital is an important component in total household wealth**, and since the next lecture will show that **private consumption depends on private wealth**, the **theory of the housing market** developed below will also **help us to understand fluctuations in private consumption.**

■ We start by considering the **production function of the construction sector**. For concreteness, suppose that the construction of new housing, I^H , is given by the production function:

$$I^H = AX^\beta, 0 < \beta < 1 \quad 10.17$$

where X is a **composite input factor** (to be specified below), and A is a **constant** which depends on the **productive capacity of the construction sector**. The assumption that the parameter β is less than 1 implies that, over the time horizon we are considering, **production is subject to diminishing returns to scale**.

■ For simplicity, we assume that **construction firms combine labour L and building materials Q in fixed proportions**. Specifically, each unit of the composite input X includes a units of labour and b units of materials:

$$L = aX, Q = bX \quad 10.18$$

- If W is the wage rate and p^Q is the price of materials, it follows from (10.18) that the **price P of a unit of the composite input X** is equal to

$$P = aW + bp^Q \quad 10.19$$

- We will refer to P as “the **construction cost index**”. If p^H is the market price of a unit of **housing**, the **sales revenue** of the representative construction firm will be $p^H I^H$, and its **profits**, Π , will be

$$\Pi = p^H I^H - PX = p^H I^H - P(I^H/A)^{1/\beta} \quad 10.20$$

- In deriving the second equality in (10.20), we have solved (10.17) for X and substituted the solution into the expression for profits. **Taking the housing price p^H and the input price P as given, the construction firm chooses its level of activity I^H with the purpose of maximizing its profit.** (We might also assume that the firm maximizes its market value. This would give the same results, but via a more cumbersome procedure.)

■ According to (10.20), the **first-order condition for profit maximization**, $d\Pi/dI^H = 0$, implies:

$$p^H - \overbrace{\frac{P}{\beta A} \left(\frac{I^H}{A} \right)^{(1-\beta)/\beta}}^{d(pX)/dI^H \text{ = marginal construction cost}} = 0 \Leftrightarrow I^H = k \cdot \left(\frac{p^H}{P} \right)^{\beta/(1-\beta)}, \quad k \equiv \beta^{\beta/(1-\beta)} A^{1/(1-\beta)}. \quad 10.21$$

■ Equation (10.21) is the **supply curve for the construction sector**. It is seen to be derived from the fact that **profit-maximizing construction firms will push construction activity to the point where the marginal construction cost equals the market price of a housing unit**.

■ The **relative price variable** p^H/P is an analogue of Tobin's q . Thus, since $0 < \beta < 1$, Eq. (10.21) says that **housing investment** I^H will be larger the higher the q -ratio of the

housing price to the construction cost index is. (In this theory of housing investment the assumption of **diminishing returns to the composite input X** has **taken the place of the installation costs** which we included in our model of business investment.) Figure 10.14 shows that this theory of housing investment fits the facts very well.

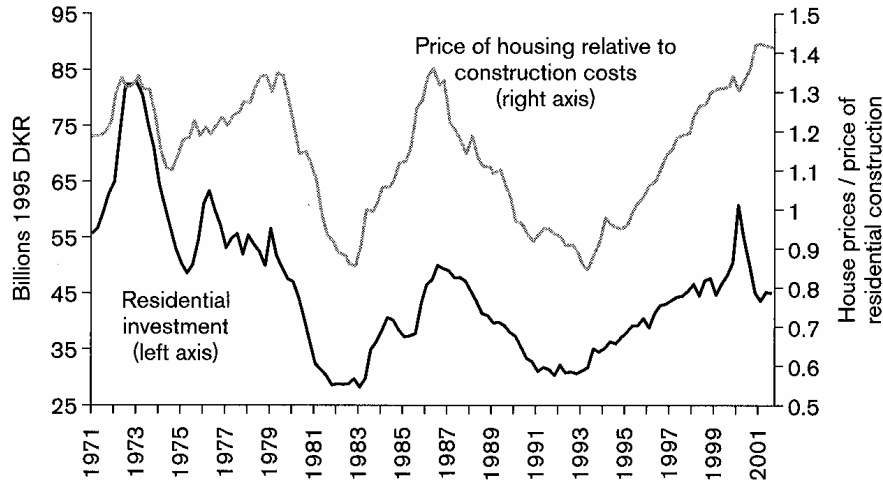


Figure 10.14: Residential investment and the price of housing relative to construction costs in Denmark

Source: MONA database, Danmarks Nationalbank.

Housing investment, interest rates and income

- Like the q -theory of business investment, our **theory of housing investment is consistent with the hypothesis that investment varies negatively with interest rates and positively with total income**. To demonstrate this, we will now develop a **theory of housing demand** in order to **explain the housing price p^H** .
- Consider a **representative consumer who has borrowed to acquire a housing stock H at the going market price p^H per unit of housing**. Suppose the consumer has to spend an amount **$\delta p^H H$ on repair and maintenance** each period to maintain the value of his house, and suppose the **interest rate on mortgage debt is r** .
- The consumer's **total cost of housing consumption** will then be $(r + \delta)p^H H$. The consumer **also consumes an amount C of nondurable goods**. If his **income is Y** , and if we abstract from savings (which will not affect our qualitative results), the **consumer's budget constraint** may then be written as:

$$C + (r + \delta)p^H H = Y \quad 10.22$$

where we have set the **price of non-durables equal to 1**.

■ The consumer wishes to **allocate his total consumption between housing and non-durables** so as to maximize his **utility U** which we assume to be given by the **Cobb-Douglas** function:

$$U = H^\eta C^{1-\eta}, \quad 0 < \eta < 1 \quad 10.23$$

■ In practice, the **consumer will derive utility from the housing service flowing from the housing stock H , and not from the housing stock as such**. The specification in (10.23) just assumes that the **housing service is proportional to the housing stock**. Using the budget constraint (10.22) to eliminate C from (10.23), we get:

$$U = H^\eta [Y - (r + \delta)p^H H]^{1-\eta} \quad 10.24$$

■ The consumer's **optimal level of housing demand** is found by **maximizing the utility function (10.24) with respect to H** . The **first-order condition $dU/dH = 0$** for the solution to this problem is:

$$\overbrace{\eta H^{\eta-1} [Y - (r + \delta)p^H H]^{1-\eta}}^{\partial U/\partial H} - (r + \delta)p^H \overbrace{(1 - \eta)H^\eta [Y - (r + \delta)p^H H]^{-\eta}}^{\partial U/\partial C} = 0 \quad 10.25$$

or

$$\frac{\partial U/\partial H}{\partial U/\partial C} = (r + \delta)p^H \quad 10.26$$

■ Equation (10.26) says that, **in the consumer's optimum situation, the marginal rate of substitution between housing and non-durables** (the left-hand side) **must equal the**

relative price of housing, $(r + \delta)p^H$. If we solve (10.25) for H , we get the **demand for housing**, now denoted as H^d :

$$H^d = \frac{\eta Y}{(r + \delta)p^H} \quad 10.27$$

- The term $(r + \delta)p^H$ in the denominator of (10.27) is sometimes referred to as the **user-cost of housing**, reflecting the **financial cost**, r , as well as the **costs of maintenance**, captured by the parameter δ which may be seen as a **depreciation rate for housing capital**.
- We see from (10.27) that **housing demand varies positively with income** and **negatively with the user-cost of housing**. Note that even if the consumer has financed the acquisition of his house by his own past savings, the user-cost should still include the interest rate r as an **opportunity cost**, since this is the **income the consumer forgoes by investing his savings in a house rather than in interest-bearing assets**.

- In practice, the **tax system also affects the user-cost of housing**. Moreover, **if the consumer expects a capital gain on his house** due to a rise in p^H , this **gain should be subtracted from the total user cost**. Here we abstract from (anticipated) capital gains for simplicity.
- While (10.27) gives the demand for housing, the **aggregate supply of housing is fixed in the short run** where the **housing stock is predetermined by the accumulated historical levels of housing investment**.
- In other words, **at the start of each period there is a given predetermined housing stock**, since the current construction activity determined by (10.21) does not add to the housing stock until the start of the next period. **In the short run, the market price of houses must therefore adjust to bring the demand for housing, H^d , in line with the existing supply, H** .
- Inserting the **equilibrium condition $H^d = H$** into (10.27) and solving for p^H , we get the **market-clearing price of houses**:

$$p^H = \frac{\eta Y}{(r + \delta)H} \quad 10.28$$

■ Figure 10.15 illustrates how the equilibrium price of houses is determined in the short run where the supply of housing is fixed at the level H_0 .

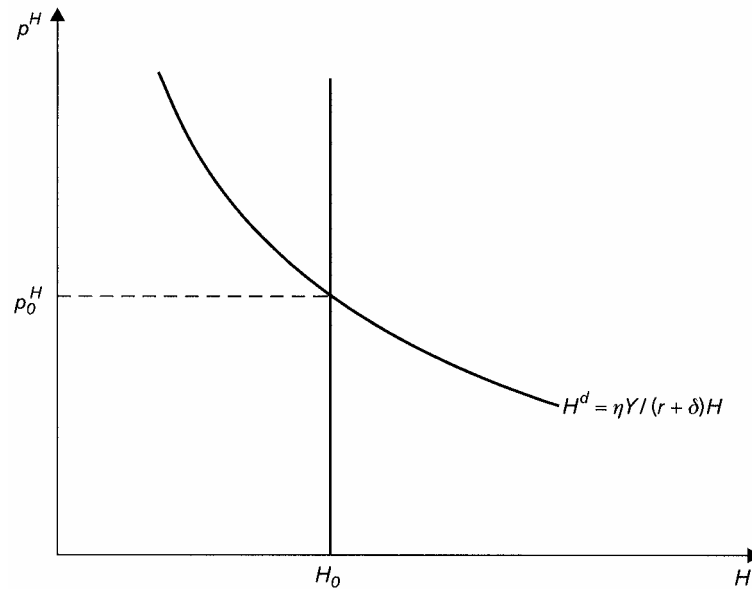


Figure 10.15: Short-run equilibrium in the housing market

- *Ceteris paribus*, a **higher preexisting housing stock will imply a lower current housing price**. We also see from (10.28) that the **housing price will be lower the higher the real interest rate r and the lower the level of income, Y** .
- Since we know from (10.21) that **current construction activity varies positively with the housing price**, we may combine (10.21) and (10.28) to get a **housing investment function** of the form

$$I^H = k \cdot \left[\frac{\eta Y}{(r + \delta)PH} \right]^{\beta/(1-\beta)}$$

or more generally:

$$I^H = h \left(\underset{(+)}{Y}, \underset{(-)}{H}, \underset{(-)}{r} \right) \tag{10.29}$$

■ The **negative impact of the interest rate on housing investment** in (10.29) is **based on the theory that a higher interest rate will, *ceteris paribus*, reduce the market price of housing**. The negatively-sloped regression line in Figure 10.16 confirms that **housing prices do in fact tend to fall when the bond interest rate goes up**, and vice versa.

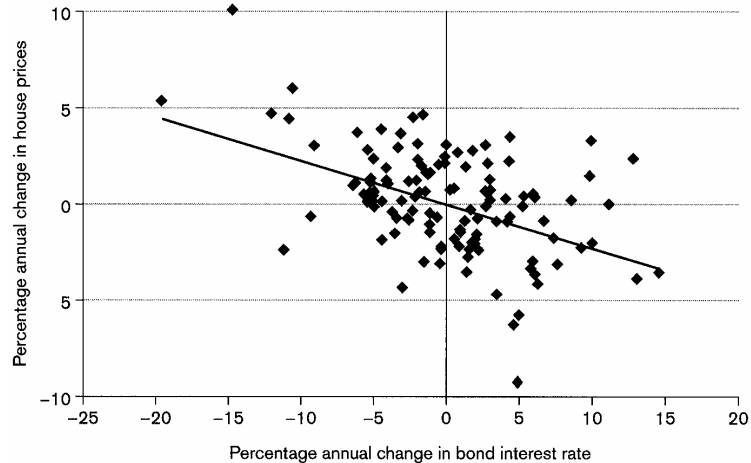


Figure 10.16: The long-term bond interest rate and house prices in Denmark

Note: Percentage changes are normalized by subtracting the average change from all observations. Quarterly data, 1975-1998.

Source: MONA database, Danmarks Nationalbank,

Housing market dynamics

■ At the aggregate level, **part of the current construction activity**, I^H , serves to **compensate for the depreciation of the existing housing stock**, δH . The housing stocks in period t and in period $t + 1$ are therefore linked by the identity

$$H_{t+1} = H_t(1 - \delta) + I_t^H \quad 10.30$$

■ Equations (10.21), (10.28) and (10.30) constitute a **simple dynamic model of the housing market**. For given values of Y and r , the predetermined housing stock, H_t , determines the housing price for period t via (10.28).

■ Given the value of P , Eq. (10.21) then determines the **current level of housing investment** I_t^H which subsequently **determines the next period's housing stock** H_{t+1} via (10.30). We

then get a **new housing price** p_{t+1}^H via (10.28) which enables us to determine I_{t+1}^H by use of (10.21), giving a **new housing stock** H_{t+2} via (10.30), and so on.

- This **dynamic process will continue until the housing price has reached a level where construction activity is just sufficient to compensate for the depreciation of the existing housing stock so that the stock of housing remains constant.**

- Thus, whereas an **upward shift in housing demand is fully absorbed by a rise in house prices in the short run, over the longer run it will cause an increase in the housing stock which will dampen the initial price increase.**

Summary

- Empirically, changes in stock prices and in housing prices tend to be followed by changes in output in the same direction. In part this reflects that higher asset prices lead to higher investment. This lecture explains the links between asset prices and investment.

- A firm seeking to maximize the wealth of its owners will choose an investment plan which maximizes the market value of the firm's assets. The value of the firm, referred to as the fundamental stock value, is the present discounted value of the expected future dividends paid out by the firm. This follows from the shareholder's arbitrage condition which says that the expected return to shareholding, consisting of dividends and capital gains on shares, must equal the return to bondholding plus an appropriate risk premium.

- When share prices reflect the fundamental value of firms, there are three possible reasons for the observed volatility of stock prices: (i) fluctuations in (the growth rate of) expected future dividends, (ii) fluctuations in the real interest rate, and (iii) fluctuations in the required risk premium on shares. There is indirect evidence that the required risk premium fluctuates quite a lot.

- The evidence suggests that the rate of return on stocks is tied to the rate of return on bonds over the long term. This accords with the view that stock prices reflect the fundamental value of firms. However, many observers believe that stock prices can sometimes deviate from fundamentals. The analysis in this lecture abstracts from such “bubbles” in stock prices.

- Increases in the firm's capital stock imply adjustment costs (installation costs), including costs of installing new machinery, costs of training workers to use the new equipment, and perhaps costs of adapting the firm's organization. These installation costs will typically increase more than proportionally to the firm's level of investment.
- The value-maximizing firm will push its investment to the point where the shareholder's capital gain from a unit increase in the firm's capital stock is just offset by the dividend he must forgo to enable the firm to purchase and install an extra unit of capital. Because the marginal installation cost is increasing in the volume of investment, this investment rule implies that the firm's optimal level of investment will be higher, the higher a unit increase in the firm's capital stock is valued by the stock market. An increase in the ratio of stock prices to the replacement cost of the firm's assets will therefore stimulate its investment.
- The market value of stocks relative to the replacement value of the underlying business assets is referred to as Tobin's q . Our theory of investment may be summarized by saying that business investment is an increasing function of Tobin's q .

- Stock prices reflect expected future dividends which tend to be positively affected by a rise in current profits. The value of Tobin's q therefore tends to vary positively with current profits, which in turn vary positively with the output-capital ratio. Hence investment is an increasing function of current output and a decreasing function of the existing capital stock.
- *Ceteris paribus*, a rise in the real interest rate implies that expected future dividends are discounted more heavily, leading to a fall in Tobin's q via lower stock prices. Thus a higher real interest rate tends to depress investment. A rise in the required risk premium on shares, generated by more uncertainty about the future, will have a similar negative impact on investment.
- A version of the q -theory can explain investment in owner-occupied housing. When the market price of residential property increases relative to the cost of housing construction, it becomes profitable for firms in the construction sector to increase the supply of new housing units. As a consequence, housing investment (construction activity) goes up. There is strong empirical evidence in favour of this hypothesis.

■ In the short run, the market price of housing varies positively with current income and negatively with the real interest rate and with the existing housing stock. Since construction increases with the market price of housing, it follows that housing investment is an increasing function of income and wealth and a decreasing function of the real interest rate and the current housing stock.