PART III. ENDOGENOUS GROWTH

5. PRODUCTIVE EXTERNALITIES AND ENDOGENOUS GROWTH

■ Although the **Solow models** studied so far are **quite successful in accounting for many important aspects of economic growth**, they have one **major limitation**: by **treating the rate of technological change as exogenous**, they **leave the economy's long-run (steady state) growth rate unexplained**. Solow models therefore belong to the class of so-called exogenous growth theories.

■ In this part of the course we will try to answer a very **big question** that remains unanswered: **how can we explain the rate of technological change** which is the source of long-run growth in income per capita? The search for an answer to this fundamental question takes us to the modern **theory of endogenous growth** where the **long-run rate of growth in GDP per person is truly endogenous.**

■ A **model that "explains"** the long-run rate of growth in GDP per worker is one that **endogenizes the underlying rate of technical change**, that is, makes this rate depend on basic model parameters. Hence, by an **endogenous growth model** we mean a model in which the long-run growth rate of **technology depends on basic model parameters** such as the **investment** rates in physical and human capital, the **population growth** rate, or other fundamental characteristics of the economy.

■ An endogenous growth model therefore allows an analysis of **how economic policies** that affect these basic parameters will **affect long-run growth** in income per capita. This is an important, and some will say the **defining, feature of endogenous growth models**: structural economic **policy has implications for growth** in output per capita in the long run.

■ In this lecture we will study endogenous growth theory. The **models** to be presented can be divided into **two categories**. **In both categories there will be aggregate production functions involving a variable A_t** that describes "technology", but there will be **no assumption of exogenous technological progress** such as $A_{t+1} = (1 + g)A_t$, where g is exogenous.

■ One category contains **models that include an explicit description of how technological progress**, $A_{t+1} - A_t$ in period t, **is produced** through a specific production process that requires inputs of its own. Since we think of the production of technological progress as arising from **research and development**, we call such models **R&D-based models** of endogenous growth. We will **not consider them in this course**.

■ The **other category** does **not have an explicit production process for technological improvement**, but **assumes that the A_t of the individual firm depends positively on the aggregate use of capital**, or of output, because of so-called "**productive externalities**".

■ This implies that the **aggregate production function**, as opposed to that of the individual firm, **will have increasing returns to scale**. As we will see, **this will result in growth in GDP per worker** in the long run **without any exogenous technological progress being assumed**. The models in this category are referred to as **endogenous growth models based on productive externalities**, and they are the subject of this lecture.

A growth model with productive externalities

■ In Lecture 3 we **explained why growth in income per worker had to vanish** in the long run according to the **basic Solow model**. The explanation was related to **constant returns to capital and labour** and the **associated diminishing returns to capital alone**. Let us take the explanation once more in a way that is well suited to our present purposes.

■ The **production function** was $Y_t = K_t^{\alpha} L_t^{1-\alpha}$, exhibiting constant returns to K_t and L_t and **diminishing returns to K_t alone (since $\alpha < 1$)**. Consequently there were also **diminishing returns to capital per worker** in the production of output per worker: $y_t = k_t^{\alpha}$.

■ Now, **assume that there is some growth in the labour force**, say at **1 per cent per year**. **If capital also increases by 1 per cent per year**, as in the steady state of the basic Solow model, then **because of constant returns to capital and labour, output will increase by 1 per cent** per year. Hence **output per worker will be constant**.

■ **How could there possibly be growth in output per worker?** With the production function of the basic Solow model, **only if capital increases by more than 1 per cent** per year. If capital increases at a given constant rate of more than 1 per cent per year, say at 2 per cent, then each year output will also increase by more than 1 per cent and hence there will be growth in output per worker.

■ Indeed, the (approximate) **growth rate of capital per worker**, g_t^k, will be constant and equal to 1 per cent, and the (approximate) growth rate in output per worker, g_t^y, will be $g_t^y = \alpha g_t^k$. Hence, there will be a constant and positive growth rate in output per worker. However, the **formula $g_t^y = \alpha g_t^k$ already reveals the problem**.

■ Since $\alpha < 1$, the **growth rate of income per worker is smaller than the assumed growth rate of capital per worker**. Therefore the **continued constant growth rate in capital per worker cannot be sustained by savings**. What happens is that **as long as capital increases faster than labour there will be more and more capital per worker** and this implies, due to diminishing returns, that **additional units of capital per worker create less and less additional output per worker**, and hence, **less and less additional savings per worker**. As a **consequence, growth in capital, and in GDP, per worker will have to cease** in the long run.

■ This reasoning suggests that **if there were increasing returns to capital and labour**, then long-run **growth in GDP per worker would be possible without exogenous technological progress**. **If both capital and labour were increasing at a rate of 1 per cent** per year, then, simply **because of increasing returns, output would be increasing by more than 1 per cent** per year.

■ And **in this case growth would not have to cease in the long run**, since it would be **unnecessary to build up more and more capital per worker to sustain growth** and hence **diminishing returns would not be a problem**.

■ **Increasing returns to scale at the aggregate level** therefore seems to be a **potential source of endogenous growth**.

Constant returns at the firm level and increasing returns at the aggregate level

■ One **may think that the idea of an endogenous growth** model suggested by the above reasoning **is simply the basic Solow model** with the **production function of the representative firm exhibiting increasing returns**, that is, with the **two exponents on capital and labour summing up to a number greater than 1** rather than exactly 1.

■ For **two reasons** this is not an idea we will pursue. First, because of the **replication argument**, we believe that there should be **close to constant returns at the firm level to the inputs that can be replicated**.

■ Second, the **idea of competitive markets**, involving price-taking behaviour of the individual firms, is **not compatible with increasing returns at the firm level**. Here is why.

■ **Under constant returns to scale and given input prices, total costs are proportional to total output**. The reason is simply that an **increase in output requires proportional**

increases in the inputs. Hence, **under constant returns average and marginal cost are constant** and **both equal to the cost of producing one unit**, $\hat{C}$, say.

■ **Under increasing returns**, an **increase in output requires less than proportional increases in the inputs**, and therefore **average and marginal costs will be decreasing in output**. With a **Cobb-Douglas** production function with the **exponents adding up to a number greater than 1**, the **marginal cost**, $\hat{C}(Y_t)$, will be a **decreasing function**, and $\hat{C}(Y_t)$ **will go to 0 as Y_t goes to infinity**.

■ **If the firm takes the prices of inputs and outputs as given, to maximize profits it should want to produce an infinite amount of output**. In other words, **profit maximization does not imply well-defined behaviour of the individual firm**. **Price-taking behaviour and perfect competition are not compatible with increasing returns at the firm level**.

■ There is a **way to keep the assumption of constant returns at the firm level** and at the same time have **increasing returns at the aggregate level**. In our model we only have **one representative profit-maximizing firm**, but this **firm represents the aggregate behaviour of many firms** each of which is small relative to the whole economy.

■ The **firm therefore takes aggregate magnitudes such as GDP or the aggregate use of capital as given**, since it is **too small to have more than a negligible influence on aggregates**. In our model the **aggregate use of capital has to be equal to the use of capital in the single representative firm**, but we should nevertheless assume that in making its individual decisions, the representative firm takes the aggregates as given.

■ We assume that the individual **production function of the representative firm** (in its role as a small individual firm) is:

$$Y_t = (K_t^d)^\alpha (A_t L_t^d)^{1-\alpha},\ 0 < \alpha < 1 \qquad 5.1$$

where the firm takes the **labour productivity variable A_t as given** and there are **constant returns to the inputs of capital and labour**, K_t^d and L_t^d.

■ We **assume** further that the **A_t of the individual firm depends positively on the aggregate stock of capital K_t** as expressed by the constant elasticity function:

$$A_t = K_t^{\varphi},\ \varphi \geq 0 \qquad 5.2$$

■ The **special case $\varphi = 0$** will bring us back to the **basic Solow model**, but we will explain below why it may be reasonable to assume $\varphi > 0$.

■ Since the **individual firm has no influence on aggregate capital**, it **takes A_t as given**. The aggregate production results from inserting (5.2) into (5.1) and using the facts that clearing of the input markets implies $K_t^d = K_t$ and $L_t^d = L_t$, where L_t is total labour supply in period t:

$$Y_t = K_t^{\alpha}(K_t^{\varphi}L_t)^{1\text{-}\alpha} = K_t^{\alpha+\varphi(1\text{-}\alpha)}L_t^{1\text{-}\alpha} \quad 5.3$$

■ When $\varphi > 0$, the **aggregate production function in (5.3) has increasing returns because the sum of the exponents is 1 + φ (1 – α) > 1**. A doubling of both aggregate inputs implies that aggregate output is multiplied by the factor $2^{1 + \varphi(1 - \alpha)}$.

■ Since the **individual firm takes aggregate capital as given**, the **marginal products entering the "marginal product equal to real rental rate" conditions** for optimal input demands should be those that appear when A_t is taken as given. Taking partial derivatives in (5.1), we therefore have:

$$r_t = \alpha\left(\frac{K_t^d}{A_t L_t^d}\right)^{\alpha-1} \text{ and } w_t = (1-\alpha)\left(\frac{K_t^d}{A_t L_t^d}\right)^{\alpha} A_t \quad 5.4$$

in which we can insert the market-clearing conditions $K_t^d = K_t$ and $L_t^d = L_t$ as well as (5.2) to arrive at:

$$r_t = \alpha\left(\frac{K_t}{K_t^{\varphi}L_t}\right)^{\alpha-1} \text{ and } w_t = (1-\alpha)\left(\frac{K_t}{K_t^{\varphi}L_t}\right)^{\alpha} A_t \quad 5.5$$

■ You will easily verify from (5.3) and (**5.5**) that $r_tK_t = \alpha Y_t$ and $w_tL_t = (1-\alpha)Y_t$. Hence, we have achieved what we wanted: we have **constant returns at the firm level (so perfect competition can be assumed)**, we have **increasing returns at the aggregate level**, and our theory of the **functional income distribution still has the nice features of constant income shares and no pure profits**, capital's share being α.

■ For this whole construction the **assumption of a productive externality from the aggregate use of capital to labour productivity**, Eq. (5.2), **is crucial**. What could be the **motivations** for this?

■ Empirically, the **idea of increasing returns at the aggregate level is not implausible**. A survey of **empirical estimates of returns to scale in aggregate production functions** can be found, for example, in Stephanie Schmitt-Grohe, "Comparing Four Models of Aggregate Fluctuations due to Self-Fulfilling Expectations", *Journal of Economic Theory*, 72, 1997. **Estimates of the sum of our exponents**, $1 + \varphi(1 - \alpha)$, are **systematically greater than 1**, pointing to $\varphi > 0$.

■ Furthermore they **vary widely across investigations**, ranging from just above **1 to levels way above 2**. There is perhaps a tendency that **most estimates** (and the most reliable ones) should be found in the **(lower) range, 1.1–1.5**.

■ A value for the sum of exponents, $1 + \varphi(1 - \alpha)$, around 1.3, say, corresponds (with α around 1/3) to φ being around 0.45, while a value of 1.5 would mean that φ should be 0.75. This may give an indication of largest possible plausible values for φ. Taking the great uncertainty of the estimations into account, however, **one cannot completely exclude larger values for the sum of the exponents**, for instance values close to 5/3 (1.67), **The latter corresponds to a φ around 1, a possibility that will be of importance below**.

■ For the **theoretical motivations** the key phrase is **learning by doing**. The idea is that the use of **(additional) capital in an individual firm will have a direct effect on production** as expressed by the individual production function (5.1), but in addition it will have a **positive effect on the capabilities of workers who gain new knowledge by working with the new capital**.

■ The **benefit from this effect does not only accrue to the firm which increases its capital stock**, because **other firms can gain from it by "looking over their shoulders"**, and because in the longer run **employees may move between firms** and thus **bring their acquired capabilities** with them to new employers.

■ These features should explain why the additional capability **effect spills over to firms in general and hence should be modelled as an externality**. But **why should there be a capability effect** at all?

■ Here the idea is that **one way workers get more skilful and sophisticated** is through the **installation of new capital in the firms**, since **new machinery is a carrier of new technological knowledge**. Thus, **working with new capital**, learning by doing, **workers become more sophisticated**.

■ Taking this idea literally, the **capability effect should arise from accumulated gross investment rather than from the stock of capital**, but we may **let the latter approximate all the gross investment undertaken in the past** (in this connection ignoring depreciation).

■ The important **idea of productive externalities due to learning by doing** comes from a famous article by the economist and Nobel Prize winner **Kenneth J. Arrow**, "The Economic Implications of Learning by Doing", *Review of Economic Studies*, 29, 1962. Arrow constructs a **model that involves a distinction between old and new capital** and in which the idea of a capability effect from the use of new capital is perhaps better placed.

<u>The complete model</u>

■ The above explanations have focused on the crucial and only new feature of the growth model we are constructing. In all other respects the **model is just like the basic Solow model**. We can therefore write down the complete model.

■ Equation (5.6) below is the **individual production function of the representative firm taking A_t as given**, but with the equilibrium conditions of the factor markets, $K_t^d = K_t$ and $L_t^d = L_t$, inserted. Equation (5.7) states the assumption that the **labour productivity variable**, A_t, potentially (if $\varphi > 0$) **depends on aggregate capital because of learning-by-doing productive externalities**. The two last equations, (5.8) and (5.9), describe **capital accumulation** and **population growth**, respectively, and are essentially unchanged from the basic Solow model:

$Y_t = (K_t)^\alpha (A_t L_t)^{1-\alpha}, \; 0 < \alpha < 1$ 5.6

$A_t = K_t^\varphi, \; \varphi \geq 0,$ 5.7

$K_{t+1} = sY_t + (1 - \delta)K_t, \; 0 < s < 1, \; 0 < \delta < 1$ 5.8

$L_{t+1} = (1 + n)L_t, \; n > -1$ 5.9

■ We have chosen not to restate the expressions for the real factor prices this time, but whenever needed they can be taken from (5.5) above. **From given initial values, K_0 and L_0**, of the **state variables**, the **model will determine the evolutions of all the endogenous variables** Y_t, K_t, L_t and A_t, and hence, using (5.5), of the real factor prices.

■ **Sometimes it is assumed** in similar models that the **external learning-by-doing effect on labour productivity really arises from total production rather than from capital use**. This would amount to the formulation, $A_t = Y_t^{\varphi}$, rather than (5.7) above, the remaining model being the same.

■ This **alternative model works qualitatively exactly** as the one above. We could also have chosen to let the external effect from capital use (or production) be affecting total factor productivity rather than labour-augmenting productivity, A_t, without any effect on our conclusions.

Semi-endogenous growth

■ As mentioned, in the special case of $\varphi = 0$, the model above is just the basic Solow model. We will therefore **assume $\varphi > 0$** in all that follows, and the **aggregate production function** (5.3) will then have **increasing returns to K_t and L_t**. Note from (5.3) that **if $\varphi < 1$, then there will be diminishing returns to capital alone**, since $\alpha + \varphi(1 - \alpha)$ will be smaller than 1, while **$\varphi = 1$, the aggregate production function will have constant returns to K_t alone**. Whether there are **diminishing returns or constant returns to capital alone** turns out to make an **important difference**. We will first consider the case $\varphi < 1$.

The law of motion

■ Assuming $\varphi < 1$, we can again analyse the model in terms of the **technology-adjusted variables**, $\tilde{k}_t \equiv k_t / A_t \equiv K_t /(A_t L_t)$ and $\tilde{y}_t \equiv y_t / A_t \equiv Y_t /(A_t L_t)$, but note that **$A_t$ is no longer growing at an exogenous rate**. Instead, the **evolution of A_t is endogenous** and **depends**, through (5.2), **on how aggregate capital evolves**.

■ It follows straightforwardly from the production function (5.6) that:

$$\tilde{y}_t = \tilde{k}_t^{\alpha} \quad 5.10$$

and (5.7) implies that:

$$\frac{A_{t+1}}{A_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\varphi} \quad 5.11$$

■ These two equations will be used in the following. From the definition of $\tilde{k}_t$ we have

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = \frac{\frac{K_{t+1}}{K_t}}{\frac{A_{t+1}}{A_t}\frac{L_{t+1}}{L_t}} = \frac{\frac{K_{t+1}}{K_t}}{\left(\frac{K_{t+1}}{K_t}\right)^{\varphi}\frac{L_{t+1}}{L_t}} = \frac{1}{1+n}\left(\frac{K_{t+1}}{K_t}\right)^{1-\varphi} \qquad 5.12$$

where we have used (5.11) and (5.9). Inserting that $K_{t+1} = sY_t + (1-\delta)K_t$, we get:

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = \frac{1}{1+n}\left(\frac{sY_t + (1-\delta)K_t}{K_t}\right)^{1-\varphi} = \frac{1}{1+n}\left(\frac{s\tilde{y}_t}{\tilde{k}_t} + (1-\delta)\right)^{1-\varphi} = \frac{1}{1+n}\left(s\tilde{k}_t^{\alpha-1} + (1-\delta)\right)^{1-\varphi}$$

where we have used (5.10) for the latter equality. Multiplying on both sides by $\tilde{k}_t$ gives the **transition equation**:

$$\tilde{k}_{t+1} = \frac{1}{1+n}\tilde{k}_t(s\tilde{k}_t^{\alpha-1} + (1-\delta))^{1-\varphi} \qquad 5.13$$

which we can also write **in the form**:

$$\tilde{k}_{t+1} = \frac{1}{1+n}(s\tilde{k}_t(\alpha+\varphi-\alpha\varphi)/(1-\varphi) + (1-\delta)\tilde{k}_t^{1/(1-\varphi)})^{1-\varphi} \qquad 5.14$$

■ Note that **since we have assumed φ < 1**, the **exponents** in (5.14) are all **well-defined and positive**. Also note that if we set $\varphi = 0$, both of the equations (5.13) and (5.14) become the transition equation of the basic Solow model (with $B = 1$).

Convergence to steady state

■ The **transition equation** has the form of a **one-dimensional first-order difference equation** in $\tilde{k}^*$. We will establish properties of the transition equation implying that **in the long run** $\tilde{k}_t$ **converges to a specific value** $\tilde{k}^*$.

■ From either of (5.13) or (5.14), it is clear that the **transition curve passes through (0, 0)**. From (5.14) it follows that it is everywhere increasing. Further, from (5.13) follows that it **crosses the 45° line for exactly one positive value** $\tilde{k}^*$ of $\tilde{k}_t$: inserting $\tilde{k}_{t+1} = \tilde{k}_t$ into (5.13) and solving for $\tilde{k}_t$ gives

$$\tilde{k}_t = \tilde{k}^* = \left(\frac{s}{(1+n)^{1/(1-\varphi)} - (1-\delta)} \right)^{1/(1-\alpha)} \qquad 5.15$$

verifying a unique positive intersection if $(1 + n)^{1/(1-\varphi)} > 1 - \delta$. The latter is realistically assumed (note that it is implied by the realistic assumption, $n + \delta > 0$).

■ Finally, **for convergence** to $\widetilde{k}^*$ it is **important that the transition curve intersects the 45° line** at $\widetilde{k}^*$ **from above**, as shown in Figure 5.1.

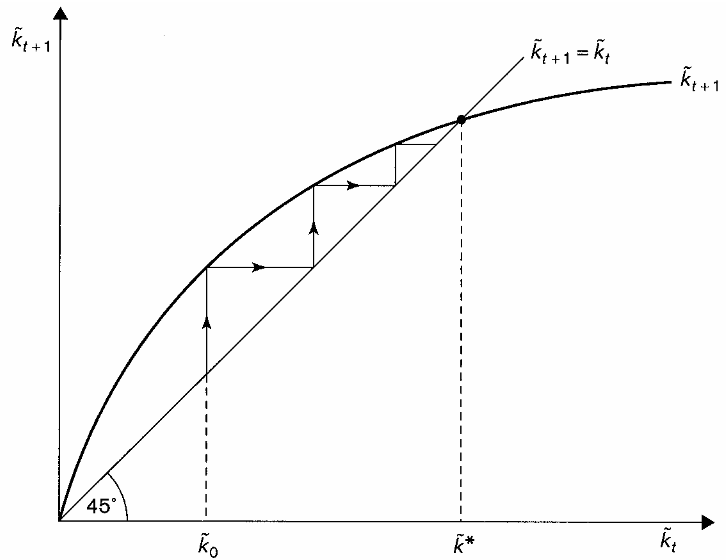


Figure 5.1: The transition diagram of the model of semi-endogenous growth

■ To show this one can differentiate the transition function given in (5.13) with respect to $\tilde{k}_t$, insert $\tilde{k}^*$ for $\tilde{k}_t$ to find the slope of the transition curve at $\tilde{k}^*$, and then derive the condition for this slope being less than 1. You will find the condition $(1 + n)^{1/(1-\varphi)} > 1 - \delta$, which is exactly the one we have just assumed.

■ The properties just established and illustrated in Figure 5.1 imply that in the long run $\tilde{k}_t$ will converge to $\tilde{k}^*$. Consequently, $\tilde{y}_t = \tilde{k}_t^{\alpha}$ will converge to the associated:

$$\tilde{y}^* = \left(\frac{s}{(1+n)^{1/(1-\varphi)} - (1-\delta)} \right)^{\alpha/(1-\alpha)} \quad 5.16$$

■ We have thus **demonstrated long-run convergence** of $\tilde{k}_t$ and $\tilde{y}_t$ to $\tilde{k}^*$ and $\tilde{y}^*$, respectively. **This defines the steady state**.

Semi-endogenous growth in steady state

■ Our **conclusions so far seem reminiscent** of those we arrived at for the **general Solow model**. The **auxiliary variables** are **defined in the same way**, $\tilde{k}_t \equiv k_t / A_t$ and $\tilde{y}_t \equiv y_t / A_t$, respectively, and we have **found again that these variables converge to constant steady state** values.

■ When they are **locked at these constant values in steady state**, both k_t and y_t **must grow at the same rate as A_t** (otherwise k_t/A_t and y_t/A_t could not be constant). **In the Solow model this sufficed for determining the steady state growth rates of k_t and y_t**. Both had to be **equal to the exogenous growth rate, g, of A_t**.

■ This time we **do not have an exogenous growth rate of A_t**, so to determine the steady state growth rate of y_t we **must determine the endogenous growth rate of A_t in steady state**.

■ This is easy to do. Consider (5.12) above. **In steady state** the left-hand side, $\tilde{k}_{t+1}/\tilde{k}_t$, **is equal to 1**. Therefore the **right-hand side must also equal 1**:

$$\frac{1}{1+n}\left(\frac{K_{t+1}}{K_t}\right)^{1-\varphi}=1 \Leftrightarrow \frac{K_{t+1}}{K_t}=(1+n)^{1/(1-\varphi)}$$

■ Using (5.11) then gives:

$$\frac{A_{t+1}}{A_t}=(1+n)^{\varphi/(1-\varphi)} \Leftrightarrow \frac{A_{t+1}-A_t}{A_t}=(1+n)^{\varphi/(1-\varphi)}-1 \equiv g_{se} \qquad 5.17$$

■ Hence, **in steady state the growth rate of A_t is g_{se}**. Note that g_{se} is determined endogenously and depends on model parameters.

■ **In steady state the growth rates of** both **capital per worker**, k_t, and **GDP per worker**, y_t, **must then also be g_{se}**. Hence from (5.17), **if the growth rate of the labour force is 0**, then the **steady state growth rate of GDP per capita is also 0**.

■ **For the growth rate of GDP per capita to be positive, a positive population growth rate is required**. To exploit the increasing returns in the aggregate production function an increasing labour force is required. The **term "semi-endogenous" growth refers to the fact that we only have (endogenous) growth in GDP per worker if there is (exogenous) population growth**.

■ Note that **in steady state the inputs of capital and labour are not growing at the same rates**. **Labour input grows at rate n**, but since $k_t \equiv K_t/L_t$ grows at the rate g_{se}, **capital** must be growing at approximately the rate $n + g_{se}$. Only if $n = 0$ are these growth rates equal.

■ **Is the endogenous growth rate g_{se} of GDP per person** derived from this model of a realistic size for **plausible** parameter values? **Annual population growth rates of around 0.5 per cent** are typical **for Western countries** and according to the empirical studies mentioned earlier, realistic values of φ could be up to 0.5–0.75. As you will easily verify from (5.17), these **parameter values result in steady state growth rates**, g_{se}, in the range **from 0.5 per cent to 1.5 per cent**, which is not far-fetched.

Structural policy for steady state

■ Let us now turn to the steady state growth path. From the definition of $\tilde{y}_t$ the steady state growth path of y_t must be $y_t^* = \tilde{y}^* A_t$. The evolution of A_t obeys $A_t = K_t^{\varphi}$, so $y_t^* = \tilde{y}^* K_t^{\varphi}$. Furthermore, **when the economy is in steady state there is a necessary link between K_t and L_t**. This follows because in steady state the variable $\tilde{k}_t$, defined as $K_t/(A_tL_t) = K_t^{1-\varphi}/L_t$, is locked at the value $\tilde{k}^*$, so in steady state:

$$K_t = (\tilde{k}^* L_t)^{1/(1-\varphi)}$$

■ Inserting this expression for K_t into the steady state growth path, $y_t^* = \tilde{y}^* K_t^{\varphi}$, for **output per worker** gives:

$$y_t^* = \tilde{y}^* (\tilde{k}^* L_t)^{\varphi/(1-\varphi)} = (\tilde{k}^*)^{\alpha+[\varphi/(1-\varphi)]} L_0^{\varphi/(1-\varphi)} (1+n)^{[\varphi/(1-\varphi)t}$$

where we have used that $\tilde{y}_t = \tilde{k}_t^{\alpha}$, and $L_t = (1 + n)^t L_0$. We can now finally insert $\tilde{k}^*$ from (5.15), and from (5.17) that $(1 + n)^{\varphi/(1-\varphi)} = 1 + g_{se}$:

$$y_t^* = \left(\frac{s}{(1+n)^{1/(1-\varphi)} - (1-\delta)} \right)^{(\alpha+[\varphi/(1-\varphi)])/(1-\alpha)} L_0^{\varphi/(1-\varphi)} (1+g_{se})^t \qquad 5.18$$

■ This gives the **steady state growth path** (y_t^*) **for output per worker**. The corresponding **steady state growth path** (c_t^*) **for consumption per worker** is obtained by multiplying by 1 – s on both sides of (5.18).

■ **Structural economic policies for steady state** must work to **affect the positions of these paths** or the **growth rates along them** (or both). With respect to the positions, the **policy implications are somewhat similar to those in the basic Solow model**.

■ For example, a higher investment rate, s, shifts the growth path for y_t upwards. Note, however, that the **golden rule value for s**, implying the highest possible position of (c_t), is now $\boldsymbol{\alpha + \varphi(1 - \alpha)}$, which is **greater than the α** we have arrived at for the basic Solow model.

■ The really **new feature** is that the **steady state growth rates of y_t and c_t depend endogenously on model parameters**: both are equal to $\boldsymbol{g_{se} = (1 + n)^{\varphi(1 - \varphi)} - 1}$. Taking φ to be a given technical parameter (not easily affected by policy) the conclusion from this model's steady state is that, **to promote long-run growth in GDP and consumption per capita, one should try to promote population growth**. There are reasons why one would **be cautious about such a policy**.

Empirics for semi-endogenous growth

■ The **model's steady state predicts that a higher growth rate in the labour** force, or the population, **should give higher growth in GDP per capita**. One way to test this prediction is to **plot average annual growth rates in GDP per worker** over some period **against average annual population growth** rates over the same period across countries.

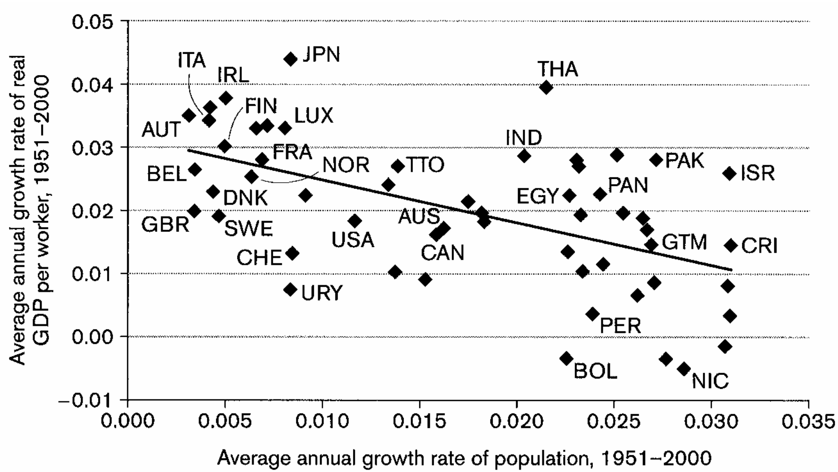


Figure 5.2: Average annual growth rate of real GDP per worker against average annual growth rate in population, 55 countries

Source: Penn World Table 6.1.

■ We find a rather **clear negative relationship between population growth and economic growth** across countries in the period considered. The OLS-estimated straight line has a significantly negative slope. This **does not necessarily mean that the model of semi-endogenous growth is wrong**, however.

■ First, it is **not clear exactly what kind of area the model covers**, a region, a country, or perhaps the (developed) world, since it is **not easy to tell how wide the external effect of the model reaches**. If the **firms in one country can "look over the shoulders" of firms in other countries**, then perhaps the model should be considered to **cover the world**. It is therefore **not clear if cross-country evidence** as reported in Figure 5.2 **is relevant**.

■ Second, the **model we consider has a steady state** and it has convergence to the steady state in the long run. **During the convergence period there will be transitory growth in addition** to the underlying (endogenous) steady state growth, and the **transitory growth depends negatively on population growth**, just as in the Solow models, and for the same reason: a **decrease in the rate of population growth shifts the steady state growth path up but reduces its slope** (see Eq. (5.18)).

■ Hence, during the transition to the new steady state growth path there will, for some time, be a **positive transitory contribution to growth arising from the lower population growth**. Furthermore, **convergence in the model we consider may be quite slow due to the presence of the productive externality**. As φ comes close to 1, convergence will be very slow. What we have found in Figure 5.2 may be interpreted as an **expression of long-lasting transitory growth** that should indeed **depend negatively on population growth** according to our model.

■ These **remarks seem to call for (even) longer run empirical evidence** that does not go across countries. Table 5.1 reports for each of the two sub-periods **1870-1930** and **1930-1990** the average annual population growth rate and the average annual growth rate of GDP per capita. **For all countries economic growth increased substantially from the early sub-period to the later one**, while for all countries but three, **population growth decreased**, for many of the countries substantially. If we consider quite long periods and "stay" within each country, the **evidence shows little sign that growth in GDP per capita should be positively related to population growth**.

■ **Still we cannot conclude that the model is wrong**. **It could be that convergence according to the model is so slow that it really takes place over periods as long as 120 years**. In that case the general picture found in Table 5.1 could be compatible with the model's transitory growth. However, **if convergence to steady state is that slow, then the model's steady state is not very descriptive for periods of interest**, so we **should consider the very slow convergence process itself** as the model's main prediction.

■ This is exactly the **perspective taken in the next section**. We will apply a specific assumption that makes the **convergence period not just very long, but infinite**. We will then consider the model's infinite, outside steady state behaviour as an approximation of a very long-lasting convergence process.

Table 5.1: Average annual growth rates in population and real GDP per capita in 17 industrialized countries, 1870-1990

	Average annual population growth rate (%)		Average annual growth rate of GDP per capita (%)	
	1870–1930	1930–1990	1870–1930	1930–1990
Australia	2.3	1.6	0.4	2.0
Austria	0.7	0.2	1.1	2.6
Belgium	0.8	0.4	1.0	2.1
Canada	1.7	1.6	1.7	2.4
Denmark	1.0	0.6	1.6	2.1
Finland	1.1	0.6	1.4	3.1
France	0.1	0.5	1.5	2.3
Germany	1.0	0.7	1.2	2.5
Italy	0.6	0.6	1.1	2.9
Japan	1.0	1.1	1.5	3.9
Netherlands	1.3	1.1	1.2	1.8
New Zealand	2.7	1.4	1.2	1.6
Norway	0.8	0.7	1.6	2.7
Sweden	0.6	0.6	1.4	2.5
Switzerland	0.7	0.8	1.7	2.1
UK	0.7	0.4	0.8	1.9
USA	1.9	1.2	1.5	2.1

Source: Angus Maddison, *Monitoring the World Economy 1820-1992*, Paris, OECD, 1995.

Endogenous growth

■ What is the analytical expression for the rate of convergence for $\tilde{y}_t$ according to our model with $\varphi < 1$? It can be shown that for the case where population growth is small, $n \approx 0$, the rate is $\lambda \approx (1 - \alpha)(1 - \varphi)\delta$. This verifies the claim that a l**arge φ less than 1 implies very slow convergence**.

■ Below **we will indeed set $n = 0$**, since we now want to investigate the possibility that over periods of interest, such as centuries, **population growth is not the factor that causes economic growth**. The size of the labour force is then constant, equal to L, say. The idea is to **consider the growth model with $\varphi = 1$**. This **gives a zero rate of convergence** and hence an everlasting convergence process. We consider the **everlasting convergence** resulting from $\varphi = 1$ to be of interest because it **approximates the very long-lasting convergence** that would result from a φ that is large, but still less than unity.

The AK model

■ In the analysis in the previous section the term $1 - \varphi$ appeared in many places, including in some denominators, and **several expressions would be meaningless for $\varphi = 1$**. Hence we **cannot simply set $\varphi = 1$** in all the equations above. For what we did above, on the other hand, it was not important whether φ was smaller than or equal to 1, so the expressions for the **real factor prices** we found there and the **associated theory of income distribution are also relevant when $\varphi = 1$**. With $L_t = L$ and $\varphi = 1$ we get from (5.5) that:

$$r_t = r \equiv \alpha L^{1-\alpha} \text{ and } w_t = (1-\alpha)K_t/L^{\alpha} \qquad 5.19$$

so the **real interest rate**, $r - \delta$, **is constant** and the **wage rate**, w_t, **evolves proportionally to K_t**, and hence to $k_t \equiv K_t/L$.

■ In addition to the above expressions for the real factor prices the **model can be condensed to the two equations**:

$$Y_t = K_t L^{1-\alpha} \equiv AK_t \qquad 5.20$$

$$K_{t+1} = sY_t + (1 - \delta)K_t \qquad 5.21$$

where the **first equation is the aggregate production function** resulting from (5.6) and (5.7), or directly from (5.3), with $\varphi = 1$, while the **second equation is the usual capital accumulation equation** repeated from (5.8). The model's last equation, (5.9), has been replaced by $L_t = L$. Note that in (5.20) we have **used the definition** $\boldsymbol{A} \equiv L^{1-\alpha}$. This is a bit of an **abuse of notation**, but it should cause **no confusion** if we denote $L^{1-\alpha}$ by an $\boldsymbol{A}$ **without a subscript** $\boldsymbol{t}$. We **apply this notation because** the model we consider is so often referred to as the "**AK model**".

Growth according to the AK model

■ It is easy to see that the **model above can result in permanent growth in GDP per worker**. Dividing both sides of the production function (5.20) by L, in order to transform variables into **per worker terms**, gives $y_t = Ak_t$. Dividing also both sides of the capital accumulation equation (5.21) by L, and inserting Ak_t for y_t then gives the **transition equation**:

$$k_{t+1} = (sA + 1 - \delta)k_t \qquad 5.22$$

■ Subtracting k_t from both sides gives the **Solow equation**:

$$k_{t+1} - k_t = sAk_t - \delta k \qquad 5.23$$

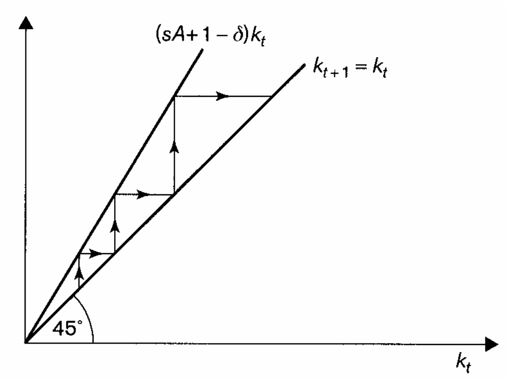
and then dividing both sides by k_t gives the **modified Solow equation**:

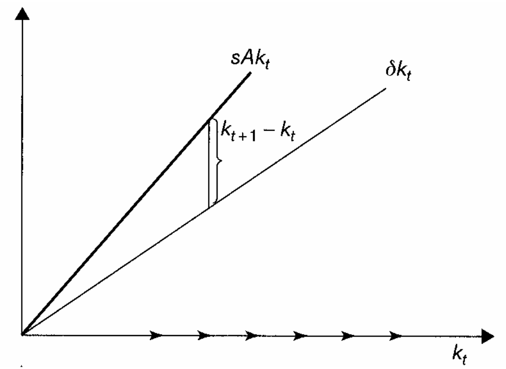
$$\frac{k_{t+1}-k_t}{k_t} = sA-\delta \equiv g_e \qquad 5.24$$

■ The latter equation directly gives the **constant and endogenous growth rate, g_e, in capital per worker k_t and hence in capital, K_t**, since $k_t = K_t/L$. Furthermore, since **y_t is a constant times k_t**, the **growth rate of output per worker is g_e**, and hence the growth rate of **consumption per worker**, $c_t = (1 - s)y_t$, **must be g_e**. Finally, the **technology variable A_t** (not to be confused with the constant A ≡ $L^{1-\alpha}$) **must increase at rate g_e**, since with $\varphi = 1$, we have **$A_t = K_t$**.

■ It follows that **according to the AK model, $g_e = sA - \delta$ is the common endogenous growth rate of all the variables** we are interested in (we assume that $sA - \delta > 0$). Hence, **everlasting economic growth is possible in this model without an assumption of exogenous technological growth**. To understand this result it may be useful to draw some well-known diagrams.

■ Figure 5.3 shows the transition diagram associated with (5.22), the Solow diagram associated with (5.23), and the modified Solow diagram associated with (5.24). The diagrams illustrate that **in this model there is no steady state** and k_t (and hence y_t and Y_t) grow at a constant rate forever.





sAk_t
δk_t
$k_{t+1} - k_t$
k_t

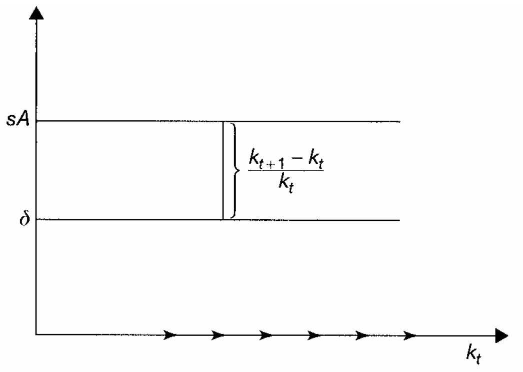


Figure 5.3: The transition diagram (top), the Solow diagram (middle), and the modified Solow diagram (bottom) of the model of endogenous growth

■ The **"curves", which are straight lines** in Figure 5.3, would **in Solow models be true curves due to diminishing returns to capital per worker** ($\alpha < 1$). The **linearity of the**

curves in the present model reflects that there are **no diminishing returns to capital at the aggregate level**. Rather there are **constant returns to capital per worker**: $y_t = Ak_t$.

■ The **"growth brake" from the Solow models**, diminishing returns to capital, is **simply no longer present** in the aggregate production function. The **source of growth is thus aggregate constant returns to the reproducible factor**, capital.

Implications for structural policy and the scale effect

■ The **main conclusion** from the model is that a **higher savings (investment) rate, *s*, gives rise to a permanently higher growth rate in GDP and consumption per worker**. This differs from our earlier conclusions since a **higher *s* no longer just gives a higher level of output per worker in the long run and a temporarily higher transitory growth rate** in the intermediate run, but it **results in a permanently higher rate of growth in output per worker**.

■ Since our **model with $\varphi = 1$ should be seen as an approximation of the semi-endogenous growth model** with a large φ smaller than 1, we should remember that strictly speaking the **correct statement is that an increase in s gives very long-lasting transitory growth in GDP per worker**. The **implications for economic policy** are obvious: policies that **stimulate savings now** give a very long-lasting boost to growth.

■ A **decrease in δ has an effect similar** to that of an increased s, but it may be more **difficult to achieve through economic policy**. **More effective aggregate investment should**, however, **lead to a lower rate of depreciation**, and a **lower rate of depreciation leads**, in the model of endogenous growth, **to a permanently higher growth rate in GDP per worker**.

■ If, somehow, a government can take actions that **make net investment more "effective"**, it can perhaps take advantage of this effect. This may be most **relevant for countries where the government is heavily involved in production** so that a large **part of investment is not driven by private incentives**, but decided upon by government bureaucracies.

■ Historically this method of making investment decisions has often led to **inefficient and rapidly worn out (sometimes even useless) investment**. By relying more on private, profit-motivated incentives in investment decisions, such a country could probably attain higher prosperity and growth in the long run, although there may be transition costs involved in changing from one type of economic system to another.

■ Policies for more **effective investment are not only relevant in connection with endogenous growth**. **In the Solow model a lower depreciation rate results in a higher level of GDP per capita in the long run**, which is also important.

■ Note the general tendency that **parameter changes that give a long-run *level* effect on GDP per worker in the Solow models give a long-run effect on the *rate of change* in GDP per worker in the model of endogenous growth**.

■ There is one **odd feature of the AK model** which casts doubt on any policy recommendation derived from it. Remember that $A \equiv L^{1-\alpha}$. Hence the **growth rate** $g_e = sA - \delta$

is higher the larger the constant population size *L* is, and the growth rate would increase if population increased.

■ This is the so-called "**scale effect**", which is rather **controversial**. Empirical **evidence does not support the hypothesis that larger countries should have larger long-run growth rates**, but again it is **not clear what geographical area the model covers**. Perhaps it is the world, but then during the last 200 years the **world population has increased rapidly while economic growth rates have not shown a similarly strong increase**.

■ It is **possible to get rid of the scale effect**. Assume that the **productive externality**, which is the driving force of endogenous growth in this lecture, **arises from capital per worker rather than from capital itself**, so that Eq. (5.7) in our model should be replaced by:

$$A_t = \left(\frac{K_t}{L_t}\right)^{\varphi}$$

■ You will easily verify that **with $\varphi = 1$ the aggregate production function becomes $Y_t = K_t$**, that is, the ***A*** **(without subscript *t*) is equal to 1 and independent of *L***.

■ Consequently the **endogenous growth rate of the model** will be $g_e = s - \delta$, and the **scale effect has been eliminated**. However, **aggregate production (not production in the individual firm) is now completely insensitive to labour input**. The **positive effect on production of a higher labour input**, L_t, arising **at the firm level is exactly offset by the negative external effect of a lower K_t/L_t at the aggregate level** (when $\varphi = 1$).

■ **Either endogenous growth models have the unattractive scale effect or they assume, unattractively, that labour inputs are unproductive at the aggregate level**. There is no way around this problem.

<u>Empirics for endogenous growth</u>

■ As mentioned, a **main prediction** of the model of endogenous growth is the **positive influence of the savings or investment rate on the long-run growth** rate of GDP per worker. Let us put this prediction to a test by considering cross-country evidence. Figure 5.4 plots **average annual growth rates**, g^i, from 1960 to 2000 **against average investment rates**, g^i, over the same period across countries i.

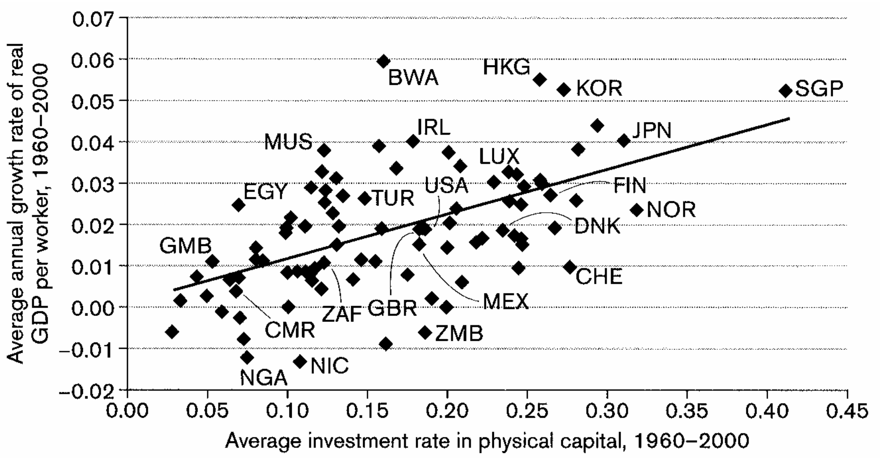


Figure 5.4: Average annual growth rate of real GDP per worker against average investment rate in physical capital, 90 countries

Source: Penn World Table 6.1.

■ The figure shows a **quite good and tight positive relationship**. This is a main reason why economists **take the theories of endogenous growth seriously** (still remembering that these should be seen as approximations of theories of very long-lasting transitory growth). Yet the **positive relationship does not prove that the idea of endogenous growth is correct**. As we have argued several times, it is **not clear whether the model covers a country or the world**, so cross-country evidence may not be the appropriate kind of empirics to study.

■ Furthermore, **in no way does Figure 5.4 contradict the traditional Solow models** according to which an **increase in the savings rate will imply a (higher) transitory growth in excess of the underlying exogenous growth**. Perhaps the figure just shows that **countries with a high average s^i** over the period considered typically **have experienced increases in s^i and therefore have had relatively high transitory growth** rates. However, the next section will provide a reason for indeed viewing Figure 5.4 as indicating endogenous growth.

Exogenous versus endogenous growth

■ In this lecture we have studied **two distinct and alternative types of endogenous growth**, **semi-endogenous** and **endogenous** growth, both fundamentally **caused by productive externalities**.

■ The presence of a **positive, but not too strong productive externality**, $0 < \varphi < 1$, gave rise to a model of **semi-endogenous growth**. This **model had convergence to a steady state** in the long-run, and therefore it had the "**convergence property**": **growth is faster the further below steady state the economy is**. Furthermore, in **steady state economic growth was explained by growth in the labour force**, and the **long-run growth rate of GDP per worker was higher the higher the rate of population growth**.

■ A **strong productive externality, $\varphi = 1$**, resulted in a **model of (genuinely) endogenous growth**. This model had **no steady state and hence no convergence to one** (or rather, in the appropriate model interpretation, very slow convergence to one). Therefore the model **did**

not have the convergence property. **Growth in GDP per capita could occur without population growth, but there was a scale effect**: a larger labour force gave a higher rate of economic growth, and a **fixed population growth would imply an exploding economic growth rate**.

■ **Both the model of semi-endogenous growth and the model of endogenous growth delivered explanations of economic growth**: the growth rate of GDP per worker depended on basic structural model parameters.

■ In the previous lectures we have studied **models of exogenous growth**. All these models had convergence to a steady state and the convergence property. **In the long run the growth rate of GDP per worker was either independent of or negatively affected by the population growth** rate (the latter in the **realistic presence of natural resources**). In these models **permanent growth in GDP per worker was a reflection of exogenous technological growth**, so the **models did not really explain long-run economic growth**.

■ The basic **mechanism for endogenous growth** in this lecture was **productive externalities**. The **technological growth** arising from these came as an **unintended by-product of economic activity**. **No agents were concerned with deliberately producing technological progress**.

■ **In the real world** we know that a lot of **research and development** activities intended to create technological progress are **undertaken in private companies as well as at universities**. Therefore, perhaps the **endogenous growth models in which technological progress is the outcome of an explicit and deliberate** production activity will be more convincing. We will not consider them here.

■ However, the **simple externality-based models of endogenous growth** considered in this lecture **share enough features with the more advanced endogenous growth models** to be representative for endogenous growth theory. Therefore we can already present some main **arguments from an ongoing and fascinating debate among growth theorists**. The issue in this debate is **whether the economic growth** we see in the real world **is best understood by**

exogenous growth models or by endogenous growth models. Here are some main arguments from the debate.

Explaining growth

■ Everybody agrees that it is **good to have explanations of long-run economic growth**, and since the endogenous growth models deliver much more of an explanation than exogenous growth models, this is certainly an **argument in favour of endogenous growth models**.

■ However, it could be that **at the present stage of knowledge the existing endogenous growth models were not convincing**, so one would have to **rely on exogenous growth models for understanding growth**, although the understanding would be limited by **technological progress being unexplained**. Whether the growth explanations and other features of the endogenous growth models are sufficiently convincing is mainly an **empirical matter**, and some arguments presented below may be important for deciding this.

The knife-edge argument

■ Strictly speaking the **model of (genuinely) endogenous growth only pertains to a zero probability, knife-edge case (φ = 1)** which is uninteresting. Endogenous growth models are sometimes criticized on such grounds. However, as we have carefully argued, the **model should rightly be interpreted as an approximation of a wider case (φ slightly smaller than 1)**, which does not have zero probability.

■ On the face of it, this criticism is therefore not valid. The issue is really whether the relevant parameter (here φ) can realistically be assumed to be so close to a limiting value (here 1) that the model of endogenous growth can be seen as a good approximation. This is again an empirical matter. For the externality parameter φ considered in this lecture, empirical **evidence seems to point to positive values of at most up to 0.75**, which speaks **in favour of models of semi-endogenous growth**.

Population growth, economic growth and the scale effect

■ **If the knife-edge property of the endogenous growth model does not represent a serious objection** to it, the **scale effect does**. The **feature that an increasing population implies an increasing growth rate in GDP per capita is simply not realistic**. The presence of the **scale effect is the single most important argument against models of (genuinely) endogenous growth**.

■ According to the **model of semi-endogenous growth** there should be a **positive relationship between the (constant) growth rate of GDP per capita and the (constant) population growth** rate. Based on the evidence presented in Figure 5.2 and Table 5.1, one may have a **hard time finding this feature convincing**.

■ There is, however, a **counterargument**: what Table 5.1 shows is true, but another fact is, according to this argument, more impressive and more important in a very long-run perspective. As we have seen, **many countries in the West have experienced relatively constant average annual growth rates in GDP per capita of about 2 per cent** during the last 200 years. Over the **several thousands of years before that**, the **average annual**

growth rates in GDP per capita must have been close to 0 to fit with the level of income per capita around year 1800.

■ So, **thousands of years of almost no economic growth have been succeeded by 200 years of rapid economic growth**. **Population growth has behaved basically the same way**, with the world population being **close to constant for hundreds of years up to the eighteenth century and thereafter increasing rapidly** (for these "facts" and an interesting discussion of them, we refer to Michael Kremer, "Population Growth and Technological Change: One Million B.C. to 1990", *Quarterly Journal of Economics*, 108, 1993).

■ This actually **fits with semi-endogenous growth**. Still, one may wonder if it is really the world's **population growth that has caused economic growth, or the other way round**. Furthermore, Table 5.1 shows that the **areas in the world that have experienced the high (and even increasing) economic growth have had decreasing population growth**.

■ It is **hard to believe that population growth in Africa, India and China** should have **caused economic growth in Europe, USA** and (other) parts of Asia, while it seems more plausible that **economic growth in Europe and USA has (indirectly) contributed to reduce mortality and create population growth in Africa and parts of Asia**.

■ Therefore the **empirical relationship between population growth and economic growth seems to be mostly in favour of the models of exogenous growth**, which predict a **negative relationship when natural resources are included** in the model.

Convergence

■ We saw in Lecture 2 that focusing on **countries that can be assumed to be structurally similar**, such as the OECD countries, there is a **clear tendency** for the countries that are initially **most behind to grow the fastest** and thus catch up to the initially richest. This pattern is even more clear between **states in the USA**.

■ We saw that **if one controls for structural differences** between countries in a way suggested by exogenous growth models, then even for the countries of the world, the **initially**

poorest tend to grow the fastest. There is thus **evidence in support of convergence** among countries in the real world.

■ In the world of models the **convergence among countries arises naturally in models with** (an appropriate form of) **convergence to a steady state**. As the steady state is approached (from below), growth becomes slower and slower. Two **different countries with similar characteristics will converge to the same steady state**, so the country starting out with the lowest GDP per person will grow the fastest. The empirically observed **convergence between countries is thus most naturally explained by models of exogenous or semi-endogenous growth**.

■ The **endogenous growth model** considered in this lecture, on the other hand, **did not have convergence to a steady state** and showed **no tendency for the initial position to be important for subsequent growth**. The observed **convergence thus contradicts the model** and seems to speak against endogenous growth. It must be added, however, that rather natural **modifications of the endogenous growth models can take account of convergence**, for example, **by introducing a gradual technological transmission between countries** into the framework.

■ The **convergence between countries** observed in the real world does **not deliver a decisive answer to what kind of growth model does the best job**.

Growth rates and investment rates

■ The single **empirical regularity** that speaks most strongly **in favour of models of (genuinely) endogenous growth** is the one illustrated in Figure 5.4: the clear **positive empirical relationship between investment rates and growth rates**. The endogenous growth model predicts that a **higher savings or investment rate should give a higher growth** rate, and indeed high investment rates do go hand in hand with high growth rates.

■ **In the models of exogenous or semi-endogenous growth** a **higher savings rate does not give a higher growth rate in the long run**, but it **does so in the intermediate run** due to transitory growth. The **positive association between investment rates and growth** rates

could thus be a result of transitory growth, and therefore it **does not necessarily contradict the exogenous or semi-endogenous growth models**.

■ However, **in models of exogenous growth a higher savings** or investment rate, s, **could not possibly give a higher growth rate in the technological variable,** A_t, exactly **because this growth rate is exogenous**.

■ **In the endogenous growth model a higher** s **will give a higher growth rate in** A_t since the growth rate of A_t is $sA - \delta$, and in the semi-endogenous growth model a higher s gives a higher transitory growth in A_t. It is thus of interest to find out **whether investment rates are also positively related to growth rates in** A_t.

■ This is indeed an idea pursued in an article by the two economists Ben S. Bernanke and Refet S. Gurkaynak ("Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously", NBER Working paper 8365, July 2001). One can **get an estimate of the average annual growth rate in the productivity variable** A_t, by growth accounting. In their article, Bernanke and Gurkaynak first do growth accounting for each of a large number of countries to get estimates of average annual growth rates of A_t. They then **plot these rates against average investment** rates across countries. Using the data of Bernanke and Gurkaynak the picture in Figure 5.5 emerges.

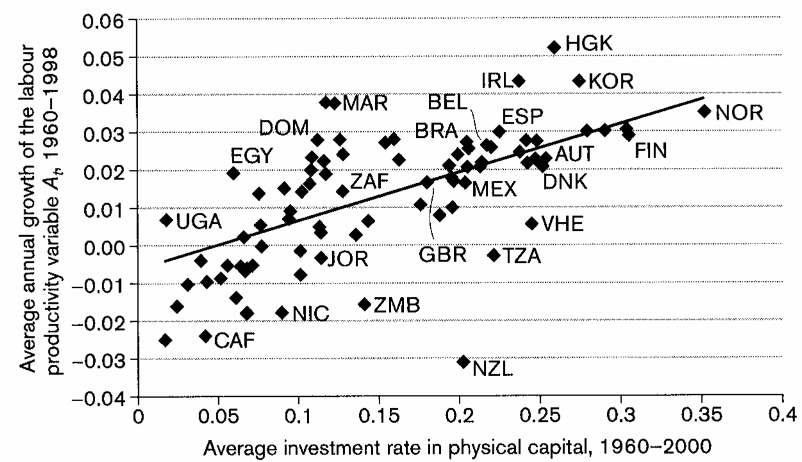


Figure 5.5: Average annual rate of labour-augmenting technological progress against average investment rate in physical capital, 84 countries

Sources: Penn World Table 6.1 and data set for Bernanke and Gurkaynak (2001).

■ There is **clearly a positive relationship**. What is shown in Figure 5.5 is the **strongest empirical argument in favour of endogenous, or perhaps semi-endogenous, growth models** that we know of.

■ The **debate** from which we have tried to present a few arguments is **not settled**. Rather it is **ongoing**. The **empirical results linking investment rates to growth rates** are sufficiently convincing to make the **endogenous growth research** programme, and the existing models, **highly promising**.

■ But a **serious challenge to endogenous growth theory**, as it stands, comes **from the empirical relationship between population growth rates and economic growth rates**, which seems, if anything, to be a **negative one**. In contrast, **semi-endogenous growth models predict a positive effect of population growth on economic growth**, while

endogenous growth models even **predict that a constant positive population growth** rate should **give an exploding growth rate in GDP per capita**.

Summary

■ In endogenous growth theory the growth rate of technology, which underlies the long-run growth in GDP per worker, is endogenous: it depends on basic behavioural model parameters and therefore on structural policies that affect these parameters.

■ According to the replication argument, the individual firm's production function should exhibit constant returns to the inputs of capital and labour at any given technological level. If there are productive externalities from the aggregate stock of capital (or from aggregate production) to labour productivity or total factor productivity in the individual firm, then there can be constant returns to capital and labour at the firm level and increasing returns to capital and labour at the aggregate level. With productive externalities one can therefore maintain the convenient assumption of perfect competition and the associated theory of income distribution, and at the same time have a potential source of endogenous growth in the model.

■ There are theoretical as well as empirical motivations for productive externalities and aggregate increasing returns. The theoretical arguments are associated with learning by doing: by being involved with (new) capital, or with production in general, workers obtain better skills. Empirical motivations come, for instance, from attempts at estimating aggregate Cobb-Douglas production functions where the sum of exponents is often found to be considerably above one.

■ Building on the idea of productive externalities, we formulated in this lecture a base model with the following features:

a. The representative firm was assumed to produce output (GDP) from the inputs of labour and capital according to a usual Cobb-Douglas production function with constant returns to capital and labour and a labour-augmenting productivity variable A_t.

b. The labour productivity variable of the representative firm depended on the aggregate stock of capital as given by a function, $A_t = K_t^{\varphi}$, $\varphi \geq 0$. Since the firm should be thought of as small relative to the entire economy, it took K_t, and hence A_t, as given.
c. Capital accumulated from savings the usual way, and the (gross) savings or investment rate was a given constant. The labour force was assumed to grow at a given rate, n.
d. The real prices of capital and labour were determined by the marginal products, computed for a given level of the productivity variable.

■ It makes a qualitative difference in this model if the strength of the external effects is below, equal to, or above 1: $\varphi < 1$ leads to semi-endogenous growth, $\varphi = 1$ leads to (truly) endogenous growth, and $\varphi > 1$ leads to explosive growth and basically an ill-behaved model.

■ In the case $\varphi < 1$, the model implies convergence of capital per effective worker and of output per effective worker to well-defined steady state levels. In steady state there was a common constant growth rate for output per worker and for technology. This growth rate depended in a specific way on φ and n, and it was strictly positive if and only if both of φ and n were strictly positive. The feature that growth in the labour force is required for economic growth motivates the term "semi" in semi-endogenous growth. The intuition is that the increasing returns driving the (semi-)endogenous growth can only be exploited if some input increases by itself. Given that the labour force grows, the capital stock will grow by a rate higher than n.

■ For the policy implications of the model of semi-endogenous growth it is important that both the level of, and the growth rate along, the growth path for output (and consumption) per worker depend on behavioural model parameters. The levels of these paths are positively influenced by the rate of saving and investment and negatively influenced by the population growth rate. This speaks for policies to raise investment rates and dampen population growth. On the other hand, the growth rate along the steady state growth path was positively influenced by population growth, speaking for policies to promote population growth, since in the long run the positive growth effect will outweigh the negative level effect.

■ Empirically it seems hard to find a clear positive association between population growth and growth in income per capita, although there are both pros and cons in the debate over whether growth in the labour force really promotes growth in output per worker. Given that this issue is unsettled, one would be cautious recommending policies to promote population growth, since it seems relatively certain that higher population growth will erode the stock of capital per worker and increase the pressure on scarce natural resources, thereby lowering the level of the basic growth path, whereas the positive effect on the growth rate along this path seems doubtful.

■ The negative association between population growth and economic growth found in the data does not contradict the model of semi-endogenous growth *per se* since, if convergence to steady state is very slow, the negative level effect of higher population growth will dominate the positive growth effect for a long time. In this case, however, the steady state itself will not be very descriptive of the economy, rather the convergence process will be the relevant thing to study. The empirical evidence therefore suggests looking at the model with very slow convergence, which obtains as φ tends to 1.

■ We took this idea to the extreme and assumed $\varphi = 1$, at the same time assuming a constant labour force. This gave us the so-called AK model. This model has constant returns to the reproducible factor, capital. It therefore implies a common and constant positive growth rate of capital per worker and output per worker, without any assumption of exogenous technological progress and without growth in the labour force being required. This is why the model's growth is called "truly" endogenous. The everlasting balanced growth described by the model can be seen as approximating the very long lasting transitory growth that would result if φ were smaller than, but close to 1. Importantly the endogenous growth rate depended positively on the rate of saving and investment.

■ The main policy implication of the AK model was therefore that policies to promote saving and investment become even more attractive, now not only because of their level effects, but also for their permanent (or very long lasting) effect on economic growth.

■ Empirically one finds across countries a rather strong positive relationship between, on the one hand, investment rates and, on the other hand, long-run growth rates in output per worker and in estimates of the productivity variable A_t. This is in very nice accordance with models of (truly) endogenous growth. On the other hand these models are plagued by an empirically implausible scale effect: according to endogenous growth models, a larger labour force gives higher economic growth and a growing labour force gives explosive economic growth. These features are highly counterfactual.

■ We closed this lecture by discussing the arguments for and against the theories of exogenous, semi-endogenous and endogenous growth. The scale effect implied by truly endogenous growth models speak against these models and in favour of models of semi-endogenous growth. Compared to endogenous growth models, models of exogenous growth seem easier to reconcile with the empirical evidence on convergence and on the relationship between population growth and economic growth. Nevertheless, endogenous growth models have made an interesting and promising contribution to the theory of economic growth, because of their ability to account for the observed positive relationship between investment rates and productivity growth, and because they have forced growth theorists to think harder about the forces underlying technological change.